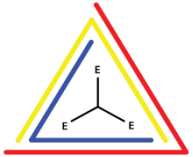




A T M E

College of Engineering



Department of EEE  
Emitting Elite Energy

# Industrial Drives And Applications–18EE741

## Selection of Motor Power Rating

Main objectives of selecting and finding out **motor power rating** are-

To obtain the suitable thermal model of motor and design the machine properly.

Finding out motor duty class.

Calculating motor ratings for various classes of duty.

## **Thermal Model of Motor for Heating and Cooling**

An accurate prediction of Heating and Cooling Curves of Electrical Drives rise inside an electrical motor is very difficult owing to complex geometrical shapes and use of heterogeneous materials. Since conductivities of various materials do not differ by a large amount, a simple thermal model of the machine can be obtained by assuming machine to be a homogeneous body. Although inaccurate, such a model is good enough for a drive engineer whose job is only to select the motor rating for a given application ensuring that temperatures in various parts of motor body do not exceed the safe limits.

Let the machine, which is assumed to be a homogeneous body, and the cooling medium has following parameters at time  $t$ :

**$P_1$  = Heat developed, joules/sec or watts.**

**$P_2$  = Heat dissipated to the cooling medium, joules/sec or watts.**

**$W$  = Weight of the active parts of machine, kg.**

**$h$  = Specific heat, Joules per kg per  $^{\circ}\text{C}$ .**

**$A$  = Cooling surface,  $\text{m}^2$ .**

**$d$  = Coefficient of heat transfer or specific heat dissipation, joules/sec/ $\text{m}^2/^{\circ}\text{C}$ .**

**$\theta$  = Mean temperature rise,  $^{\circ}\text{C}$ .**

During a time increment  $dt$ , let the machine temperature rise be  $d\theta$ . Since,

Heat absorbed (Stored) in the machine = (Heat developed inside the machine – Heat dissipated to the surrounding cooling medium)

$$Whd\theta = p_1dt - p_2dt \quad (4.1)$$

$$p_2 = \theta dA \quad (4.2)$$

Substituting In Eq.(4.1) & Rearranging the terms

$$C \frac{d\theta}{dt} = p_1 - D\theta \quad (4.3)$$

$$\text{where } C = Wh \quad (4.4)$$

$$D = dA \quad (4.5)$$

C is the thermal capacity of the machine, watts/°C, and D the heat dissipation constant, watts/°C. Heat dissipation mainly occurs through convection. Typical values of d are in the range of 40 of 600 W/m<sup>2</sup>/°C. The first order differential equation (4.3) has a solution

$$\theta = \theta_{ss} + Ke^{-t/\tau} \quad (4.6)$$

$$\text{where } \theta_{ss} = \frac{P_1}{D} \quad (4.7)$$

$$\tau = \frac{C}{D} \quad (4.8)$$

Constant of integration K is obtained by substituting the temperature rise at  $t = 0$  in Eq. (4.6).

When the initial temperature rise is  $\theta_1$ , Eq. (4.6) has a solution

$$\theta = \theta_{ss}(1 - e^{-t/\tau}) + \theta_1 e^{-t/\tau} \quad (4.9)$$

$\tau$ , which has the dimension of time, is known as the heating (or thermal) time constant of the machine. In Eq. (4.9) as  $t = \infty$ ,  $\theta = \theta_{ss}$ . Thus  $\theta_{ss}$  is the steady state temperature of the machine when it is continuously heated by power  $P_1$ . At this temperature, all the heat produced in machine is dissipated to the surrounding medium.

Let the load on machine be thrown off after its temperature rise reaches a value  $\theta_2$ . Heat loss will reduce to a small value  $P'_1$  and cooling operation of the motor will begin. Let the new value of heat dissipation constant be  $D'$ . If time is measured from the instant the load is thrown off, then

$$C \frac{d\theta}{dt} = p'_1 - D'\theta \quad (4.10)$$

Solving this first order differential equation subjected to the initial condition,  $\theta = \theta_2$  at  $t = 0$ , gives

$$\theta = \theta'_{ss}(1 - e^{-t/\tau'}) + \theta_2 e^{-t/\tau'} \quad (4.11)$$

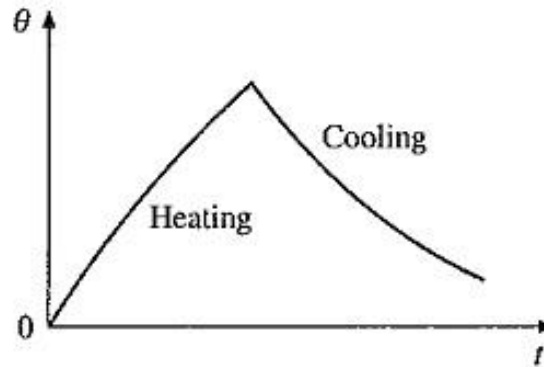
$$\theta'_{ss} = \frac{p'_1}{D'} \quad (4.12)$$

$$\tau' = \frac{C}{D'} \quad (4.13)$$

$\theta'_{ss}$  is again steady state temperature rise for new conditions of operation and  $\tau'$  is known as the **Cooling (or thermal) Time Constant** of the machine.

If motor were disconnected from the supply during Heating and Cooling Curves of Electrical Drives then  $P'_1 = \theta'_{ss} = 0$ , suggesting that the final temperature attained by the motor will be ambient temperature. Substituting in Eq. (4.11) gives

$$\theta = \theta_2 e^{-t/\tau'} \quad (4.14)$$



**Fig. 4.1** Heating and cooling curves

Figure 4.1 shows the variation of motor temperature rise with time during Heating and Cooling Curves of Electrical Drives. Thermal time constants of a motor are far larger than electrical and mechanical time constants. While electrical and mechanical time constants have a typical ranges of 1 to 100 ms and 10 ms to 10 s, the thermal time constants may vary from 10 min to couple of hours

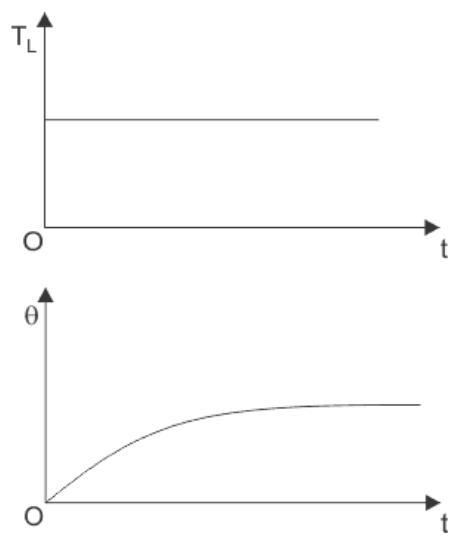
## Classes of motor duty

S1	Continuous duty	The motor works at a constant load for enough time to reach temperature equilibrium.	Paper mill drives , Compressors, Conveyors, Centrifugal pumps and Fans
S2	Short-time duty	The motor works at a constant load, but not long enough to reach temperature equilibrium. The rest periods are long enough for the motor to reach ambient temperature.	Crane drives , Drives for house hold appliances Turning bridges, Sluice gate drives, Valve drives and Machine tool drives
S3	Intermittent periodic duty	Sequential, identical run and rest cycles with constant load. Temperature equilibrium is never reached. Starting current has little effect on temperature rise.	Pressing, Cutting, Drilling machine drives.
S4	Intermittent periodic duty with starting	Sequential, identical start, run and rest cycles with constant load. Temperature equilibrium is not reached, but starting current affects temperature rise.	Metal cutting, Drilling tool drives, Drives for forklift trucks, Mine hoist etc

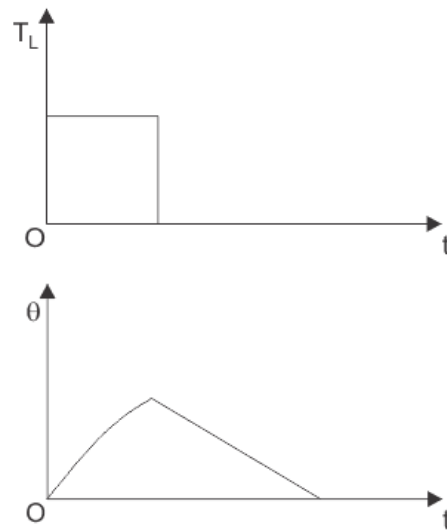
## Classes of motor duty

S5	Intermittent periodic duty with starting & electric braking	Sequential, identical cycles of starting, running at constant load and running with no load. No rest periods.	Billet mill drive, Manipulator drive, Ingot buggy drive, Screw down mechanism of blooming mill Several machine tool drives, Drives for electric suburban trains and Mine hoist
S6	Continuous operation with intermittent load	Sequential, identical cycles of running with constant load and running with no load. No rest periods.	Pressing, Cutting, Shearing and Drilling machine drives.
S7	Continuous operation with starting & electric braking	Sequential identical cycles of starting, running at constant load and electric braking. No rest periods.	The main drive of a blooming mill.
S8	Continuous operation with periodic changes in load and speed	Sequential, identical duty cycles run at constant load and given speed, then run at other constant loads and speeds. No rest periods.	All variable speed drives.

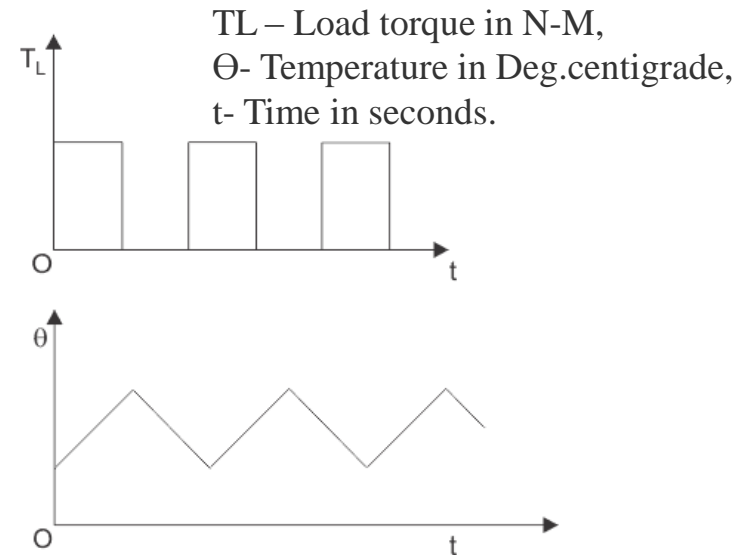




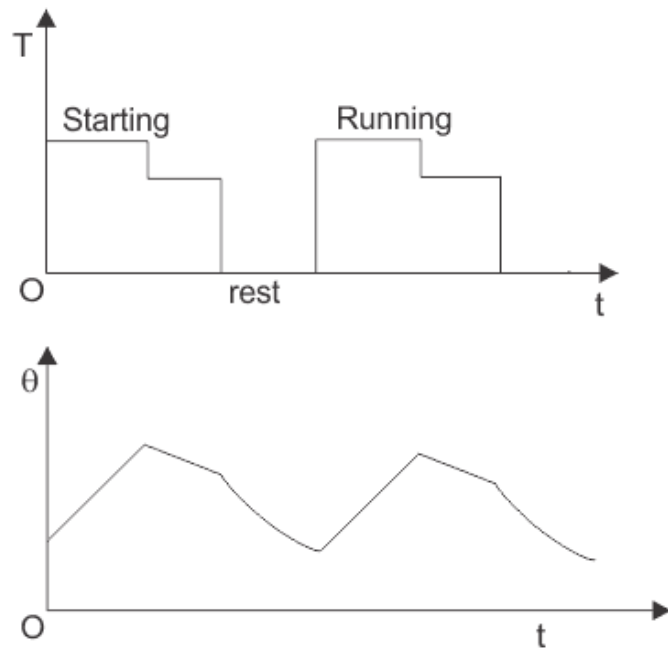
**1) continuous duty**



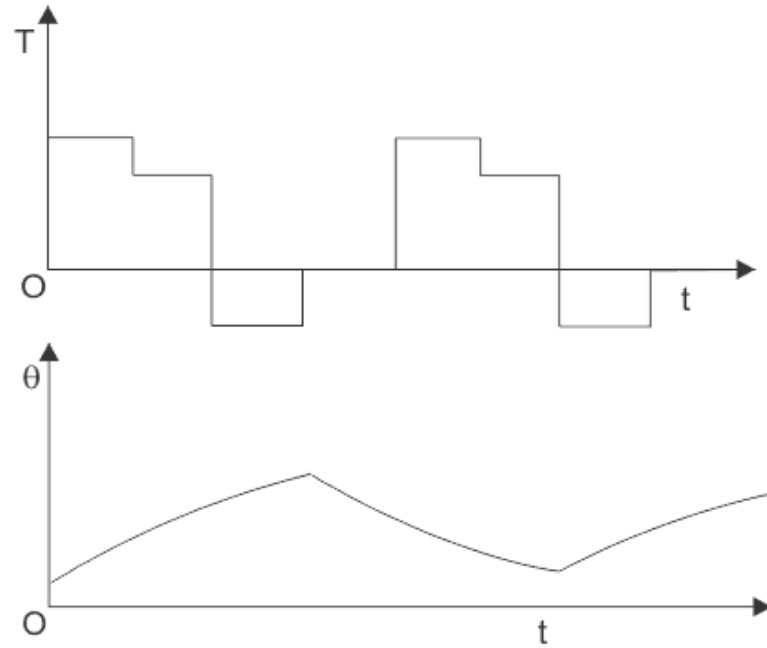
**2) short time duty**



**3) intermittent duty**



**4) Intermittent Periodic with starting**



**5) intermittent Periodic with starting & braking**

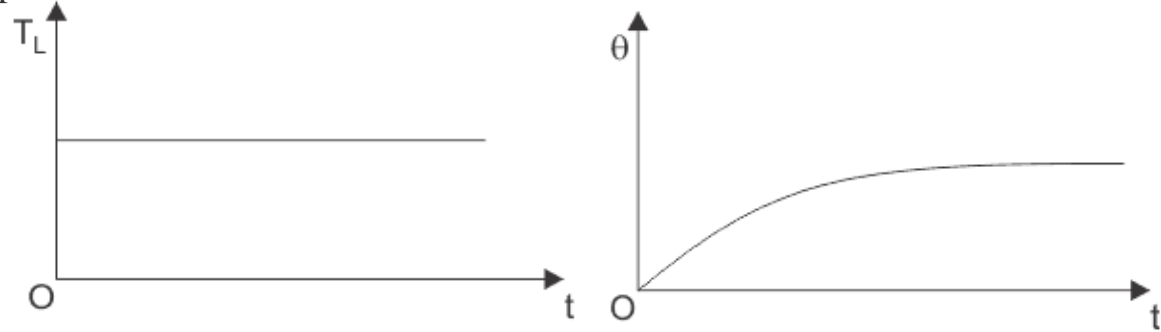
$T_L$  – Load torque in N-M,  
 $\theta$  – Temperature in Deg.centigrade,  
 $t$  – Time in seconds.

# 1. Continuous duty:

This type drive is operated continuously for a duration which is long enough to reach its steady state value of temperature.

This duty is characterized by constant motor torque and constant motor loss operation. Depicted in fig.1.

This type of duty can be accomplished by single phase/ three phase induction motors and DC shunt motors



**Examples:**  
Paper mill drives ,  
Compressors  
Conveyors,  
Centrifugal pumps and  
Fans ,

## 2. Short time duty:

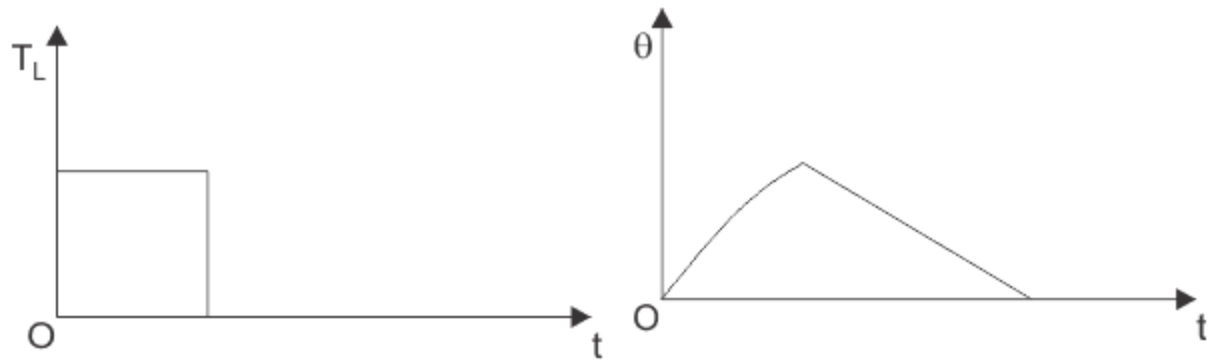
In this type drive operation, Time of operation is less than heating time constant and motor is allowed to cool off to room temperature before it is operated again.

Here the motor can be overloaded until the motor temperature reaches its permissible limit. Depicted in fig.2

This type of duty can be accomplished by single phase/ three phase induction motors and DC shunt motors, DC series motors, universal motors.

### Examples:

**Crane drives ,  
Drives for house hold appliances  
Turning bridges  
Sluice gate drives  
Valve drives and  
Machine tool drives.**



### 3. Intermittent periodic duty:

In this type drive operation, It consists of a different periods of duty cycles

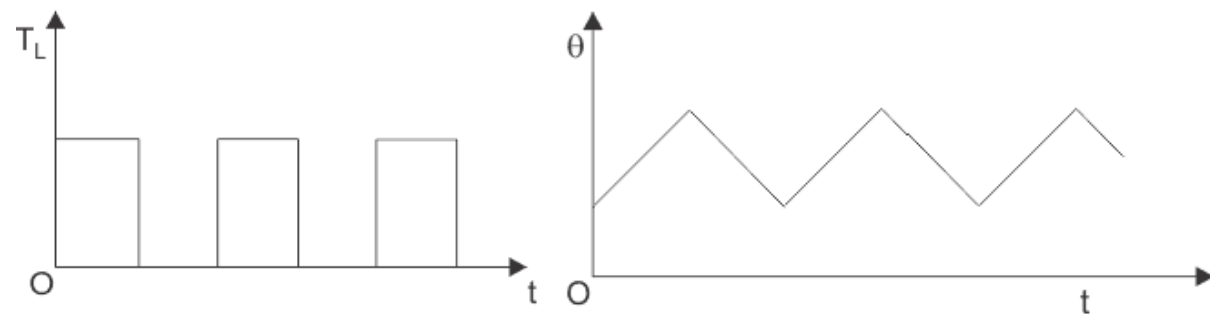
I.e. a period of rest and a period of running, a period of starting, a period of braking.

Both a running period is not enough to reach its steady state temperature and a rest period is not enough to cool off the machine to ambient temperature.

In this type drive operation, heating due to starting and braking is negligible. Depicted in fig.3

This type of duty can be accomplished by single phase/ three phase induction motors and DC shunt motors, universal motors.

**Examples:**  
**Pressing**  
**Cutting**  
**Drilling machine drives.**



#### 4. Intermittent periodic duty with starting:

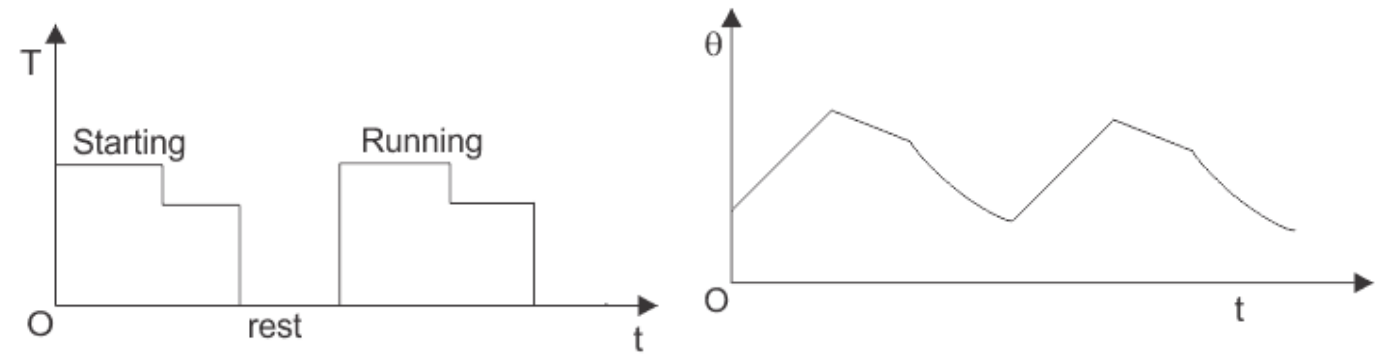
This is intermittent periodic duty where heating Due to starting can't be ignored.

It consists of a starting period; a running period, a braking period & a rest period are being too short to reach their steady state value.

In this type of drive operation, heating due to braking is negligible. Depicted in fig.4

This type of duty can be accomplished by three phase induction motors and DC series motors, DC compound motors, universal motors.

**Examples:**  
**Metal cutting,**  
**Drilling tool drives,**  
**Drives for forklift trucks,**  
**Mine hoist etc.**



## 5. Intermittent periodic duty with starting & braking:

- This is an intermittent periodic duty where heating during starting & braking can't be ignored.
- It consists of a starting period, a running period; a braking period & a rest period are being too short to reach their steady state temperature value. Depicted in fig.5
- This type of duty can be accomplished by single phase/ three phase induction motors and DC shunt motors, DC series motors, DC compound motors, universal motors.

### Examples:

**Billet mill drive**

**Manipulator drive**

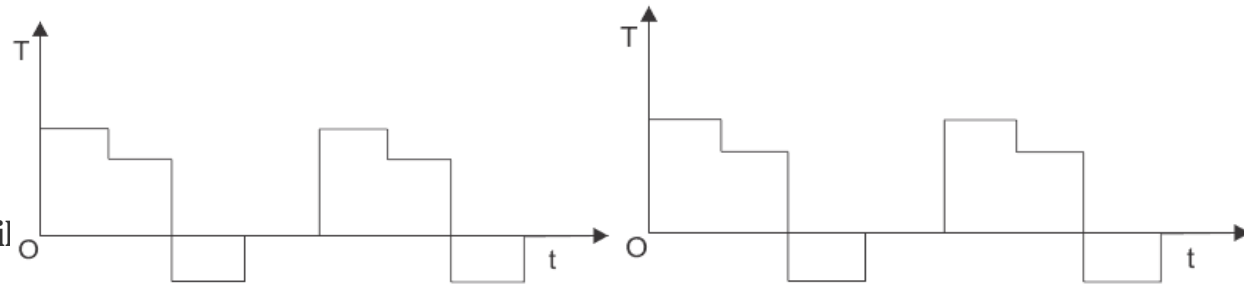
**Ingot buggy drive**

**Screw down mechanism of blooming mill**

**Several machine tool drives**

**Drives for electric suburban trains and**

**Mine hoist**



## **6. Continuous duty with intermittent periodic loading:**

- This type of drive operation consists a period of running at constant load and a period of running at no load with normal voltage to the excitation winding in separately excited machines.
- Again the load and no load periods are not enough to reach their respective temperature limits.
- This duty is distinguished from intermittent periodic duty by running at no load instead of rest period.
- This type of duty can be accomplished by single phase/ three phase induction motors and DC compound motors, universal motors.

### **Examples:**

**Pressing**

**Cutting**

**Shearing and**

**Drilling machine drives.**

## **7. Continuous duty with starting & braking:**

- It consists a period of starting, a period of running & a period of electrical braking.
- Here period of rest is negligible.
- This type of duty can be accomplished by single phase/ three phase induction motors.

**Examples:**

**The main drive of a blooming mill.**



## **8. Continuous duty with periodic speed changes:**

- It consists a period of running in a load with a particular speed and a period of running at different load with different speed which are not enough to reach their respective steady state temperatures.
- Further here is no period of rest.
- This type of duty can be accomplished by single phase/ three phase induction motors and DC series motor in traction.

### **Examples:**

All variable speed drives.

# SELECTION OF POWER RATING OF MOTORS

From the point of view of motor rating for various Classes of Motor duty cycles selection of power rating of motors can be broadly classified as:

- Continuous duty and constant load
- Fluctuating Loads (Continuous duty and variable load)
- Short time duty rating

## 1. Continuous duty and constant load

If the motor has load torque of  $T$  N-m and it is running at  $\omega$  radians/seconds, if efficiency in  $h$  , then power rating of the motor is

$$P = \frac{T\omega}{1000} KW$$

Power rating is calculated and then a motor with next higher power rating from commercially available rating is selected.

Obviously, motor speed should also match load's speed requirement .It is also necessary to check whether the motor can fulfill starting torque requirement also.

## **2. Continuous duty and variable load**

- The operating temperature of a motor should never exceed the maximum permissible temperature, because it will result in deterioration and breakdown of insulation and will shorten the service life of motors.
- It is general practice to base the motor power ratings on a standard value of temperature, say 35° c.
- Accordingly, the power given on the name plate of a motor corresponds to the power which the motor is capable of delivering without overheating at an ambient temperature of 35 c. the duty cycle is closely related to temperature and is generally taken to include the environmental factors also.
- The rating of a machine can be determined from heating considerations.
- However the motor so selected should be checked for its overload capacity and starting torque.
- This is because, the motor selected purely on the basis of heating may not be able to meet the mechanical requirements of the basis of heating may not be able to meet the mechanical requirements of the load to be driven by it.

- In many applications, the power input required for a motor is not known before hand and therefore certain difficulties arise in such cases.
- For the determination of ratings of machines whose load characteristics have not been thoroughly studied, it becomes necessary to determine the load diagram i.e., diagram shown the variation of power output versus time.
- The majority of electric machines used in drives operate continuously at a constant or only slightly variable load.
- The selection of the motor capacity for these applications is fairly simple in case the approximate constant power input is known.

The temperature of the motor changes continuously when the load is variable. On account of this, it becomes difficult to select the motor rating as per heating.

❖ The analytical study of heating becomes highly complicated if the load diagram is irregular in shape or when it has a large number of steps.

❖ Therefore it becomes extremely difficult to select the motor capacity through analysis of the load diagram due to select the motor capacity through analysis of the load diagram due to lack of accuracy of this method.

On the other hand it is not correct to select the motor according to the lowest or highest load because the motor would be overloaded in the first case and under loaded in the second case. Therefore it becomes necessary to adopt suitable methods for the determination of motor ratings

## **Methods used**

The four commonly used methods are:

- 1      Methods of average losses
- 2      Equivalent current method
- 3      Equivalent torque method
- 4      Equivalent power method

## Equivalent Current, Torque and Power Methods for Fluctuating and Intermittent Loads

This method can be employed for duties (3)-(8) (Refer Sec. **Classes of motor duty**). It is based on approximation that the actual variable motor current can be replaced by an equivalent  $I_{eq}$  which produces same losses in the motor as actual current. This equivalent current is determined as follows

Motor loss  $P_1$  consists of two components-constant loss  $P_c$ , which is independent of load and consists of core-loss and friction loss; and load dependent copper loss. Thus for a fluctuating load (Fig. a) consisting of  $n$  values of motor currents  $I_1, I_2, \dots, I_n$ , for durations  $t_1, t_2, \dots, t_n$  respectively, the equivalent current  $I_{eq}$  is given

$$P_c + I_{eq}^2 R = \frac{(P_c + I_1^2 R)t_1 + (P_c + I_2^2 R)t_2 + \dots + (P_c + I_n^2 R)t_n}{t_1 + t_2 + \dots + t_n}$$
$$P_c + I_{eq}^2 R = \frac{P_c(t_1 + t_2 + \dots + t_n)}{t_1 + t_2 + \dots + t_n} + \frac{(I_1^2 t_1 + I_2^2 t_2 + \dots + I_n^2 t_n)R}{t_1 + t_2 + \dots + t_n}$$

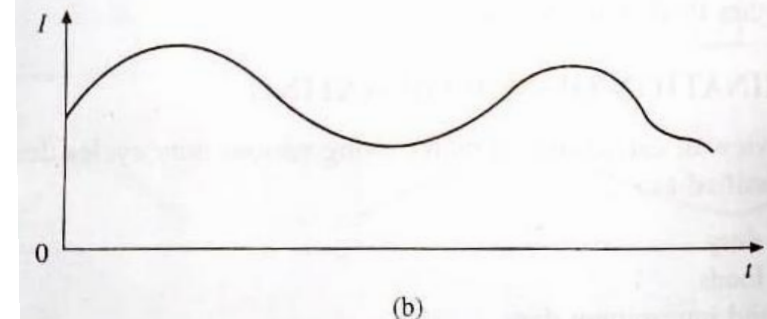
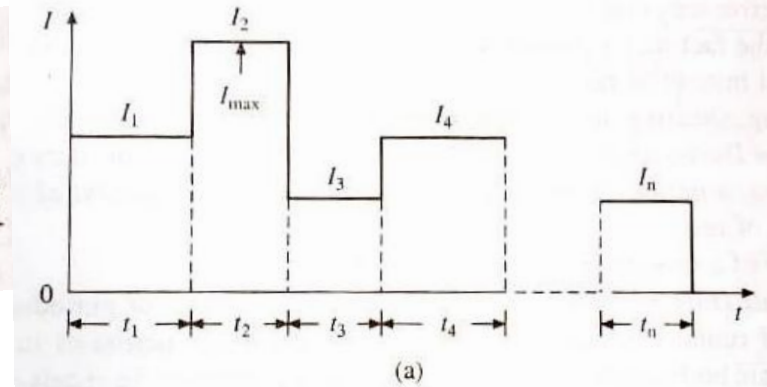


Fig. Load diagram of a fluctuating load

$$I_{eq} = \sqrt{\frac{I_1^2 t_1 + I_2^2 t_2 + \dots + I_n^2 t_n}{t_1 + t_2 + \dots + t_n}}$$

If the current varies smoothly over a period T (Fig. Load diag. of Fluctuating Load), above eq. can be written as

$$I_{eq} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

Implicit in above analysis is the assumption that heating and cooling conditions remain same.

If motor runs at a constant speed throughout this operation, heating and cooling in fact, remain same.

If speed varies, constant losses will marginally change. However, if forced ventilation is used, heating and cooling conditions can still be assumed to remain same without much loss of accuracy. In self ventilating machines, cooling conditions at low speeds will be poorer than at normal speed. Consequently above two Equations should be used with caution.

After  $I_{eq}$  is determined, a motor with next higher current rating ( $= I_{rated}$ ) from commercially available ratings is selected. Next, this rating is checked for its practical feasibility as follows

**DC Motor:** This motor can be allowed to carry larger than the rated current for a short duration. This is known as short time overload capacity of the motor. A normally designed DC machine is allowed to carry up to 2 times the rated current (3 to 3.5 times the rated current in specially designed DC machines) because for higher currents sparking between the brushes and commutator reaches an unacceptable level.

**Induction and Synchronous Motors:** In case of induction and synchronous motors, for stable operation, maximum load torque should be well within the breakdown torque of the motor. In case of induction motors with normal design, the ratio of breakdown to rated torque varies from 1.65 to 3 and for synchronous motors 2 to 2.25 (for special types up to 3.5).



When the load has high torque pulses, selection of motor rating based on this will be too large, Load equalization by mounting a flywheel on the motor shaft must then be considered

Equivalent current method assumes 'constant losses', to remain constant for all operating points. Therefore, above method should be carefully employed when these losses vary.

It is also not applicable to motors with frequency (or speed) dependent parameters of the equivalent circuit, e.g. in deep-bar and double squirrel-cage rotor motors the rotor winding resistance and reactance vary widely during starting and braking making this method inapplicable.

### **Equivalent torque method**

When torque is directly proportional to current, as for example in dc separately excited motor then  $T_{eq}$  given by below Equation and be employed to directly ascertain the motor torque rating.

$$T_{eq} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + \dots + T_n^2 t_n}{t_1 + t_2 + \dots + t_n}}$$

### **Equivalent power method**

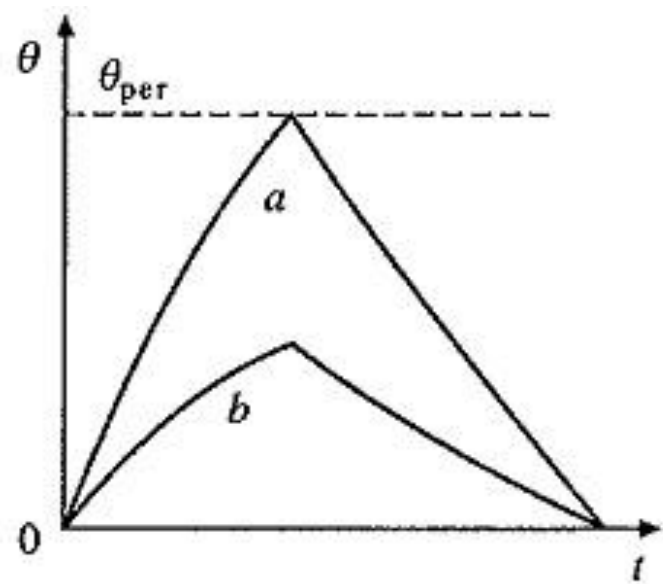
When motor operates at nearly fixed speed, its power will be directly proportional to torque.

Hence, for nearly constant speed operation, power rating of the motor can be obtained directly from

$$P_{eq} = \sqrt{\frac{P_1^2 t_1 + P_2^2 t_2 + \dots + P_n^2 t_n}{t_1 + t_2 + \dots + t_n}}$$

**Short Time Duty:**

In short time duty, time of motor operation is considerably less than the heating time constant and motor is allowed to cool down to the ambient temperature before it is required to operate again. If a motor with a continuous duty power rating of  $P_r$  is subjected to a short time duty load of magnitude  $P_r$ , then the motor temperature rise will be far below the maximum permissible value  $\theta_{per}$  and the motor will be highly underutilized (Fig. 4.4). Therefore, motor can be overloaded by a factor  $K(K > 1)$  such that the maximum temperature rise just reaches the permissible value  $\theta_{per}$  as shown in below Fig.



**Fig. 4.4**  $\theta$  vs  $t$  curves for short time duty loads:  
a—with power  $KP_r$ ; b—with power  $P_r$

When the duration of running period in a Motor Rating Various Duty Cycles with power  $KP_r$  is  $t_r$ , then

$$\theta_{\text{per}} = \theta_{\text{ss}} (1 - e^{-t_r/\tau})$$

$$\frac{\theta_{\text{ss}}}{\theta_{\text{per}}} = \frac{1}{1 - e^{-t_r/\tau}}$$

Note that  $\theta_{\text{ss}}$  is the steady state temperature rise which will be attained if motor delivers a power ( $KP_r$ ) on continuous basis, whereas the permissible temperature rise  $\theta_{\text{per}}$  is also the steady state temperature rise attained when motor operates with a power  $P_r$  on continuous basis. If the motor losses for powers  $P_r$  and ( $KP_r$ ) be  $P_{1r}$  and  $P_{1s}$ , respectively, then from above eq.

$$\frac{\theta_{\text{ss}}}{\theta_{\text{per}}} = \frac{P_{1s}}{P_{1r}} = \frac{1}{1 - e^{-t_r/\tau}}$$

$$P_{1r} = P_c + P_{\text{cu}} = P_{\text{cu}}(\alpha + 1)$$

$$\alpha = \frac{P_c}{P_{\text{cu}}}$$

and  $P_c$  is the load independent (constant) loss and  $P_{\text{cu}}$  the load dependent loss. Then

$$K = \sqrt{\frac{1 + \alpha}{1 - e^{-t_r/\tau}} - \alpha}$$

Above Equation allows the calculation of overloading factor K which can be calculated when constant and copper losses are known separately. When separately not known, total loss is assumed to be only proportional to (power)<sup>2</sup>, i.e.  $\alpha$  is assumed to be 0.

As already mentioned, K is subjected to the constraints imposed by maximum allowable current in case of dc motors and breakdown torque limitations in case of induction and synchronous motors.

1. A rolling mill driven by thyristor converter-fed dc motor operates on a speed reversing duty cycle. Motor field current is maintained constant at the rated value. Moment of inertia referred to the motor shaft is  $10,000 \text{ kg-m}^2$ . Duty cycle consists of the following intervals:

- (i) Rolling at full speed (200 rpm) and at a constant torque of 25,000 N-m for 10 sec.
- (ii) No load operation for 1 sec at full speed.
- (iii) Speed reversal from 200 to -200 rpm in 5 sec.
- (iv) No load operation for 1 sec at full speed.
- (v) Rolling at full speed and at a torque of 20,000 N-m for 15 sec.
- (vi) No load operation at full speed for 1 sec.
- (vii) Speed reversal from 200 to 200 rpm in 5 sec.
- (viii) No load operation at full speed for 1 sec.

**Determine the torque and power ratings of the motor**

- (i) Rolling at full speed (200 rpm) and at a constant torque of 25,000 N-m for 10 sec.
- (ii) No load operation for 1 sec at full speed.
- (iii) Speed reversal from 200 to -200 rpm in 5 sec.
- (iv) No load operation for 1 sec at full speed.
- (v) Rolling at full speed and at a torque of 20,000 N-m for 15 sec.
- (vi) No load operation at full speed for 1 sec.
- (vii) Speed reversal from 200 to 200 rpm in 5 sec.
- (viii) No load operation at full speed for 1 sec.

Since in a DC motor, at constant field current the torque is proportional to armature current, torque rating can be evaluated by determining the rms value of torque

$$\text{Torque during Reversal} = J * (d\omega/dt) = 10000 * [200 - (-200)] * (2\pi * 60) / 5 = 83776 \text{ N-m}$$

$$T_{eq} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + \dots + T_n^2 t_n}{t_1 + t_2 + \dots + t_n}}$$

$$T_{rms} = \sqrt{[(25000)^2 * 10 + (83776^2 * 5) * 2 + 20000^2 * 15] / 39}$$

$$T_{rms} = 47,686 \text{ N-M}$$

Maximum torque 83776 N-m is only 1.76 times  $T_{rms}$ . If motor rating is chosen to be 47686 N-m, the maximum current will be only 1.76 times the rated current. In a dc motor twice the rated current can always be allowed during transient operation. Therefore, motor can be rated equal to  $T_{rms}$ . Thus, motor torque rating

$$T_{rated} = 47686 \text{ N-m}$$

$$\text{Power Rating} = 47686 * 200 / 60 * 2\pi = 998.7 \text{ kW}$$



2. A constant speed drive has the following duty cycle.

- (i) Load rising from 0 to 400 kW: 5 min
- (ii) Uniform load of 500KW: 5 min
- (iii) Regenerative power of 400 kW returned to the supply : 4min
- (iv) Remains idle for : 2 min

Estimate power rating of the motor. Assume losses to be proportional to (power)?.

Rated power = rms value power. =  $P_{rms}$

Now the rms value of the power in interval (i) is;  $P_i = \sqrt{\frac{1}{5} \int_0^5 \left( \frac{400}{5} x \right)^2 dx} = \frac{400}{\sqrt{3}} KW$

$$P_{rms} = \sqrt{\frac{\left( \frac{400}{\sqrt{3}} \right)^2 \times 5 + 500^2 \times 5 + 400^2 \times 4}{16}}$$

$$P_{rms} = \sqrt{\frac{\left(\frac{400}{\sqrt{3}}\right)^2 \times 5 + 500^2 \times 5 + 400^2 \times 4}{16}} = 367 \text{ KW}$$

**Since Pmax = 500 KW is less than two times Prms, motor rating = 367 KW**

3. A motor has a heating time constant of 60 min and cooling time constant of 90 min. When run continuously on full load of 20 KW, the final temperature rise is 40°C.

- i) What load the motor can deliver for 10 min if this is followed by a shut down period long enough for it to cool?
- ii) If it is on an intermittent load of 10 min followed by 10 min shut down, what is the maximum value of load it can supply during the on load period?

Alpha( $\alpha$ ) is assumed to be zero, since constant  $\tau$  and copper losses are not available separately

- i) When  $\alpha = 0$ , the overloading factor **K (Short time duty)** is given by;  $K = \sqrt{\frac{1}{1 - e^{-t_r/\tau}}}$

Tau  $\tau$  which as the dimension of time known as heating (or thermal) time constant of the machine

$$K = \sqrt{\frac{1}{1 - e^{-t_r/\tau}}} = \sqrt{\frac{1}{1 - e^{-10/60}}}$$

$$K = \sqrt{\frac{1}{1 - e^{-t_r/\tau}}} = \sqrt{\frac{1}{1 - e^{-10/60}}} = 2.25$$

$$\text{Permitted load} = 2.25 \times 20\text{k} = 51 \text{ kW}$$

ii) for  $\alpha = 0$  Overload factor K ( for intermittent Periodic Duty) is given by ,

$$K = \sqrt{(\alpha + 1) \frac{1 - e^{-\{(t_r/\tau_r) + (t_s/\tau_s)\}}}{1 - e^{-t_r/\tau_r}} - \alpha}$$

$$\text{for } \alpha = 0 \quad K = \sqrt{\frac{1 - e^{-\{(t_r/\tau_r) + (t_s/\tau_s)\}}}{1 - e^{-t_r/\tau_r}}} = \sqrt{\frac{1 - e^{-\left\{\frac{10}{60} + \frac{10}{90}\right\}}}{1 - e^{-10/60}}}$$

$$K = \sqrt{\frac{1 - e^{-\{(t_r/\tau_r) + (t_s/\tau_s)\}}}{1 - e^{-t_r/\tau_r}}} = \sqrt{\frac{1 - e^{-\left\{\frac{10}{60} + \frac{10}{90}\right\}}}{1 - e^{-10/60}}} = \sqrt{\frac{0.2425}{0.1535}} = 1.257$$

**Permitted load = 1.257x20K = 25.14 KW**

4. Half hour rating of a motor is 100 KW. Heating time constant is 80 min and the maximum efficiency occurs at 70% full load. Determine the continuous rating of the motor

Given data:

Half hour rating = 100 kW

Heating time constant,  $\tau_h = 80$  min

$T_{\max}$  occurs at 70% of full load

Let  $P_c$  be constant loss and  $P$  be continuous rating of motor.

At 70% of full load ( $0.7P$ )

$$\text{Constant loss} = \text{Cu loss} \quad (1)$$

At  $P$ ,

$$\begin{aligned} \text{Cu loss} &= \left( \frac{P}{0.7P} \right)^2 P_c \\ &= \frac{P_c}{0.491} \end{aligned} \quad (2)$$

We know that

$$\begin{aligned} \alpha &= \frac{P_c}{P_{\text{copper}}} \\ &= \frac{P_c}{P_c / 0.49} \\ \alpha &= 0.49 \end{aligned} \quad (3)$$

$$K = \sqrt{\frac{1 + \alpha}{1 - e^{-t_b/\tau}}} - \alpha$$

$$= \sqrt{\frac{1 + 0.49}{1 - e^{-30/80}}} - 0.49$$

$$K = 2.0676$$

$$\text{Continuous rating} = \frac{100}{2.0676} = 48.365 \text{ kW}$$

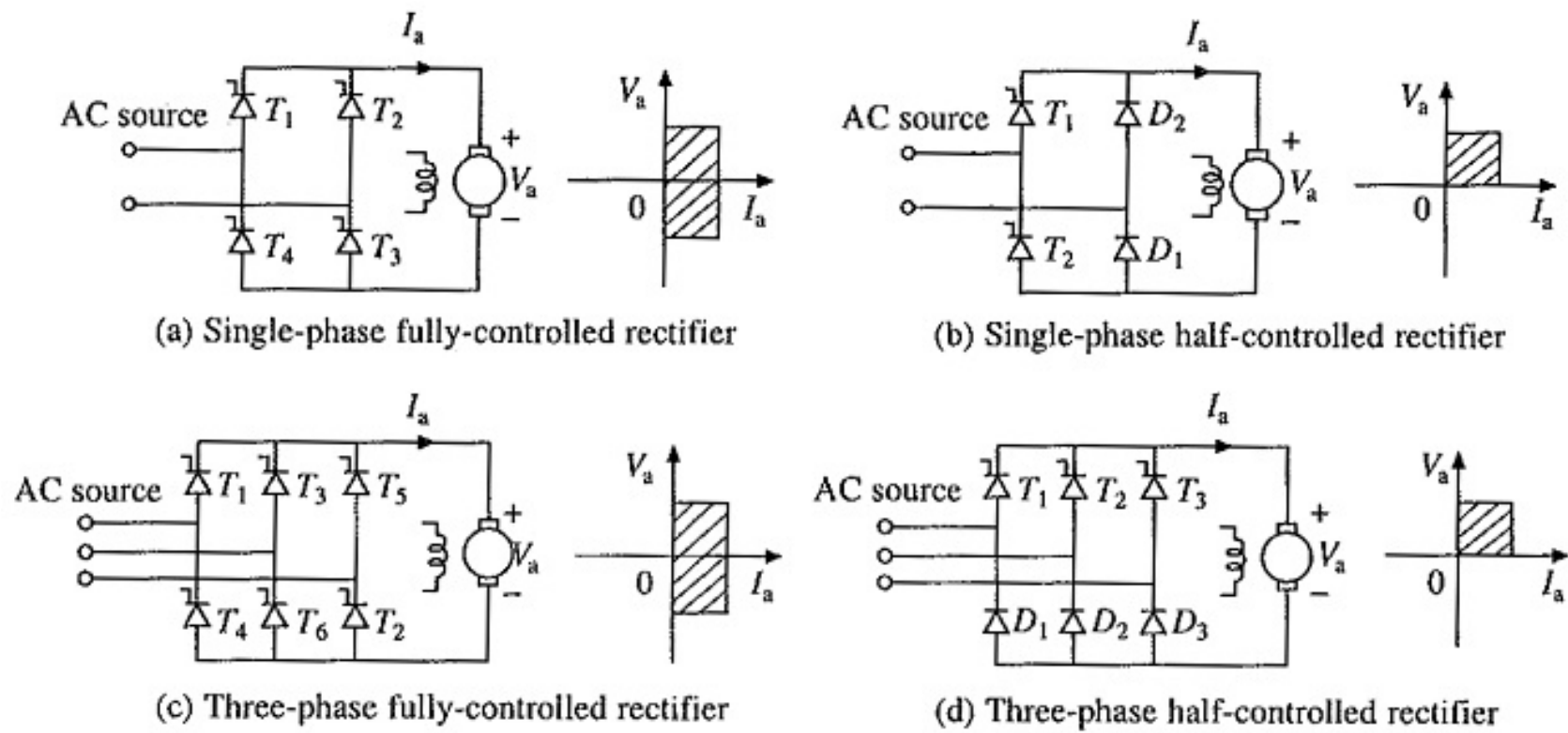
## Assignment

5. A motor operates on a periodic duty cycle in which it is clutched to its load for 10 min and declutched to run on no load for 20 min. Minimum temperature rise is  $40^{\circ}\text{C}$ . Heating and cooling time constants are equal and have a value of 60 min. When load is declutched continuously, the temperature rise is  $15^{\circ}\text{C}$ . Determine,
- i) Maximum temperature during the duty cycle.
  - ii) Temperature when the load is clutched continuously.



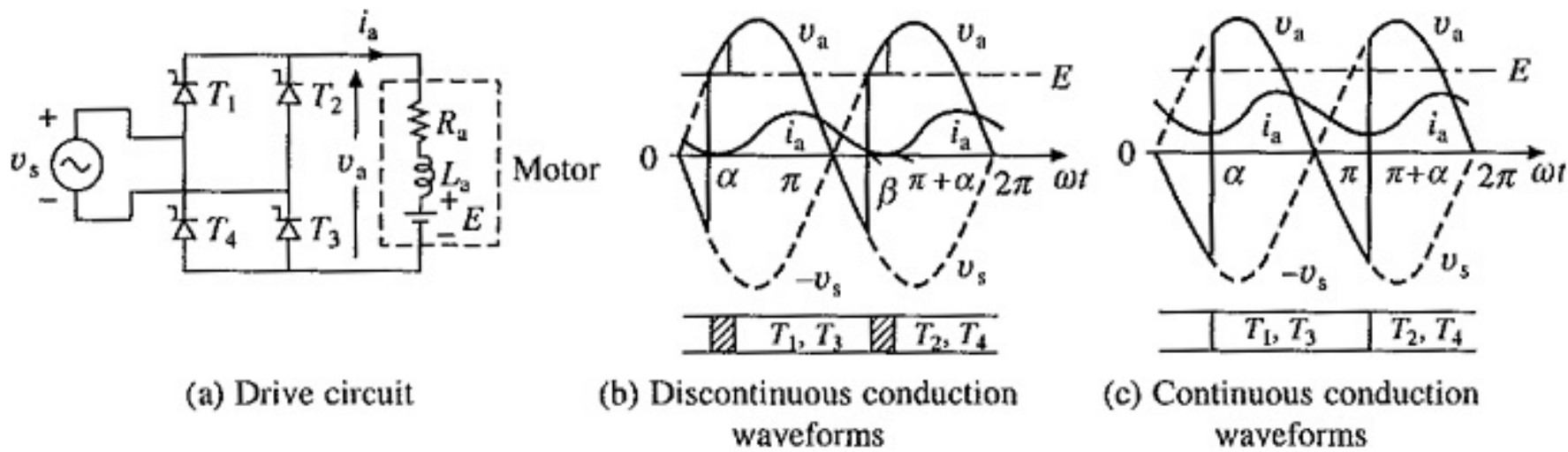
**Controlled Rectifier Fed DC Drives:**

Controlled Rectifier Fed DC Drives are used to get variable dc voltage from an ac source of fixed voltage. Controlled Rectifier Fed DC Drives are also known as Static Ward-Leonard drives



**Fig.** Single-phase and three phase controlled rectifier circuits

**Single Phase Fully Controlled Rectifier Control of DC Motor:**



**Fig.** Single-phase fully-controlled rectifier-fed dc separately excited motor

$$v_s = V_m \sin \omega t \quad ....(1)$$

**Discontinuous Conduction:**

In a Single Phase Fully Controlled Rectifier Control of DC Motor terminal voltage  $v_a$ , the drive operates in two intervals (Fig. (b)):

Duty interval ( $\alpha \leq \omega t \leq \beta$ ) when motor is connected to the source and  $v_a = v_s$ .

Zero current interval ( $\beta \leq \omega t \leq \pi + \alpha$ ) when  $i_a = 0$  and  $v_a = E$ .

**Drive operation is described by the following equations:**

# Single Phase Fully Controlled Rectifier Control of DC Motor:

## Discontinuous Conduction:

In a Single Phase Fully Controlled Rectifier Control of DC Motor terminal voltage  $v_a$ , the drive operates in two intervals (Fig. (b)):

- I. Duty interval ( $\alpha \leq \omega t \leq \beta$ ) when motor is connected to the source and  $v_a = v_s$ .
- II. Zero current interval ( $\beta \leq \omega t \leq \pi + \alpha$ ) when  $i_a = 0$  and  $v_a = E$ .

Drive operation is described by the following equations:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + E = V_m \sin \omega t, \text{ for } \alpha \leq \omega t \leq \beta \quad \dots(2)$$

$$v_a = E \quad \text{and} \quad i_a = 0 \quad \text{for} \quad \beta \leq \omega t \leq \pi + \alpha \quad \dots(3)$$

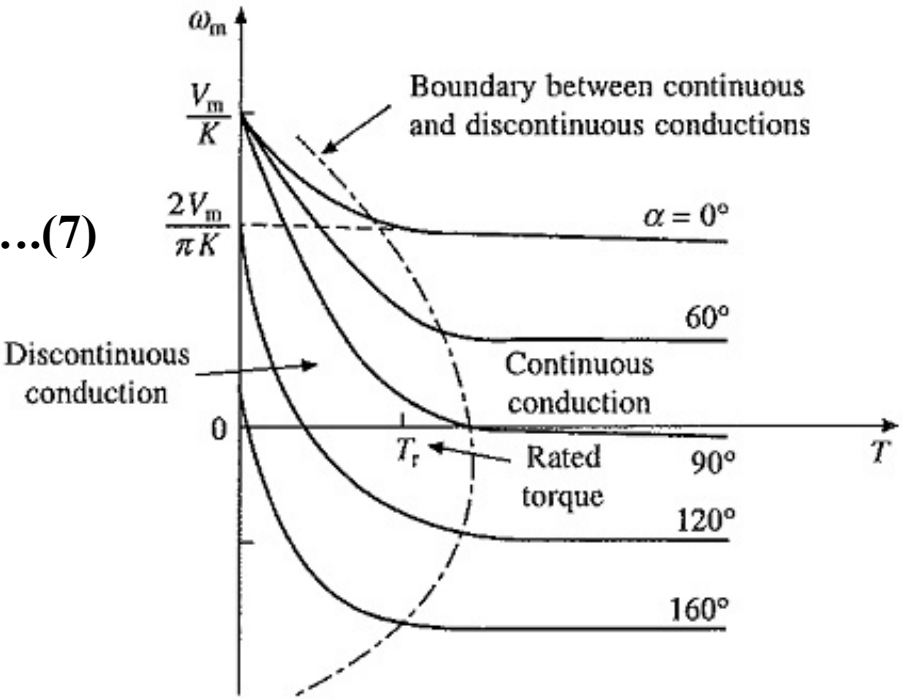
$$\text{where} \quad Z = \sqrt{R_a^2 + (\omega L_a)^2} \quad \dots(4)$$

$$\phi = \tan^{-1} (\omega L_a / R_a) \quad \dots(5)$$

Boundary between continuous and discontinuous conduction is reached when  $\beta = \pi + \alpha$ . gives the critical value of speed  $\omega_{mc}$  which separates continuous conduction from discontinuous conduction for a given  $\alpha$  as

$$\omega_m = \frac{V_m (\cos \alpha - \cos \beta)}{K (\beta - \alpha)} - \frac{\pi R_a}{K^2 (\beta - \alpha)} T \quad ....(6)$$

$$i_a (\omega t) = \frac{V_m}{Z} [\sin (\omega t - \phi) - \sin (\alpha - \phi) e^{-(\omega t - \alpha) \cot \phi}] - \frac{E}{R_a} [1 - e^{-(\omega t - \alpha) \cot \phi}], \quad \text{for } \alpha \leq \omega t \leq \beta \quad ....(7)$$



**Fig.** Speed torque characteristics of single-phase fully-controlled rectifier fed dc separately excited motor

# Single Phase Half Controlled Rectifier Control of DC Separately Excited Motor:

Single Phase Half Controlled Rectifier Control is shown in Fig. (a). In a cycle of source voltage defined by Eq. (1),  $T_1$  receives gate pulse from  $\alpha$  to  $\pi$  and  $T_2$  from  $(\pi+ \alpha)$  to  $2\pi$ . Motor terminal voltage and current waveforms for the dominant discontinuous and continuous conduction mode are shown in Figs. (b) and (c) respectively.

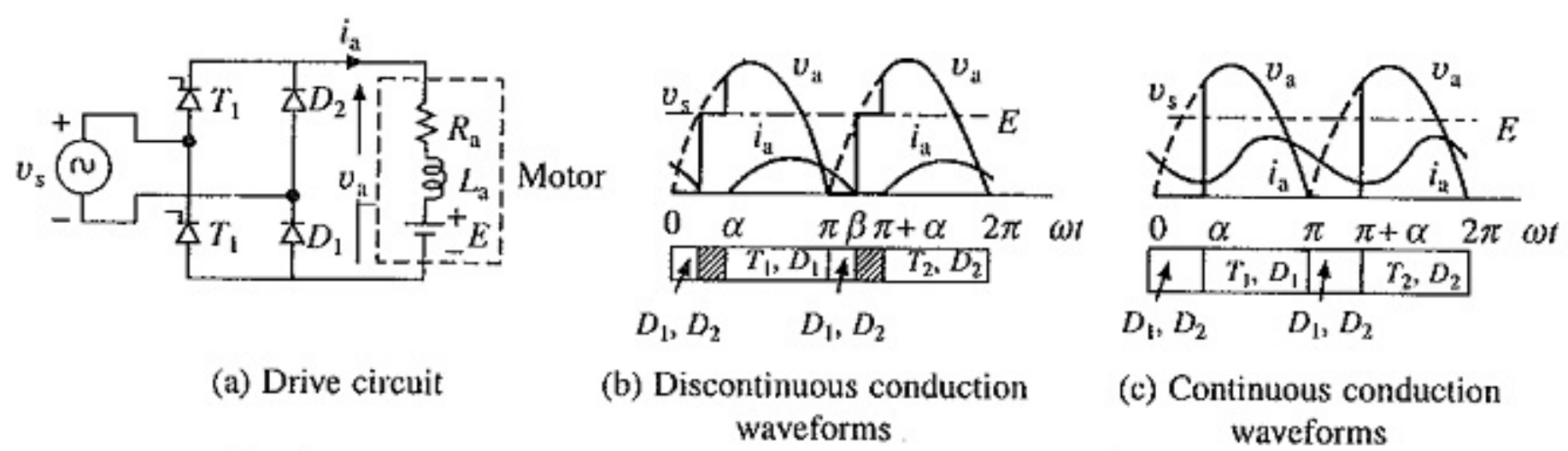


Fig. Single-phase half-controlled-rectifier fed separately excited motor

**Discontinuous Conduction:**

A cycle of motor terminal voltage consists of three intervals (Fig. 5.29(b)):

- Duty interval ( $\alpha \leq \omega t \leq \pi$ ): Armature current is given by Eq. (7). Substitution of  $\omega t = \pi$  in this equation gives  $i_a(\pi)$ .
- Freewheeling interval ( $\pi \leq \omega t \leq \beta$ ): Operation is governed by the following equation:

$$i_a R_a + L_a \frac{di_a}{dt} + E = 0 \quad \dots(8)$$

Three phase Fully Controlled Rectifier Control of DC Separately Excited Motor:

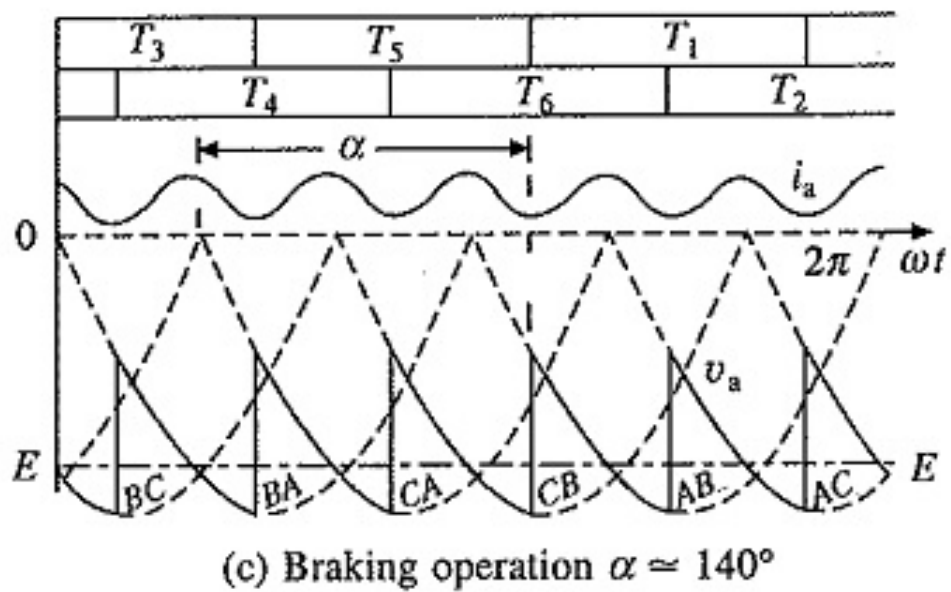
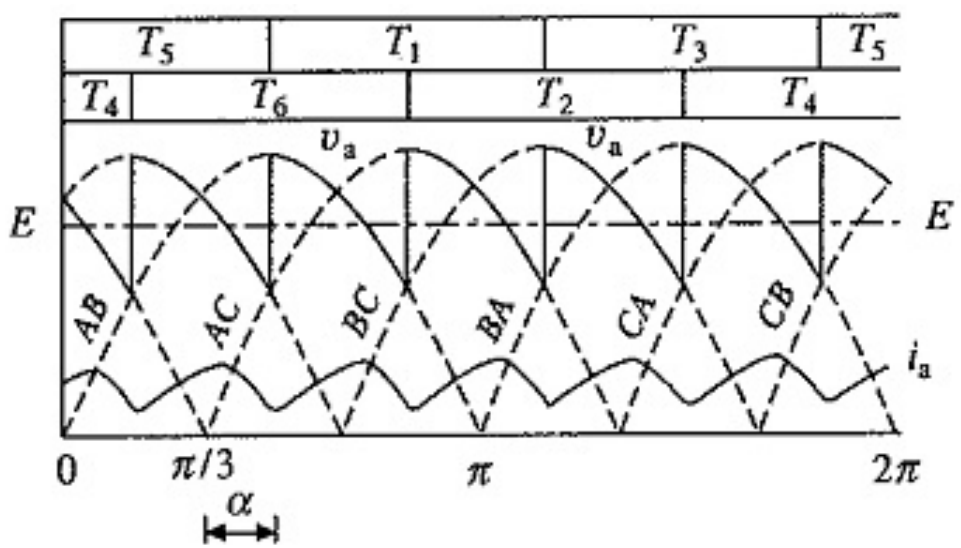
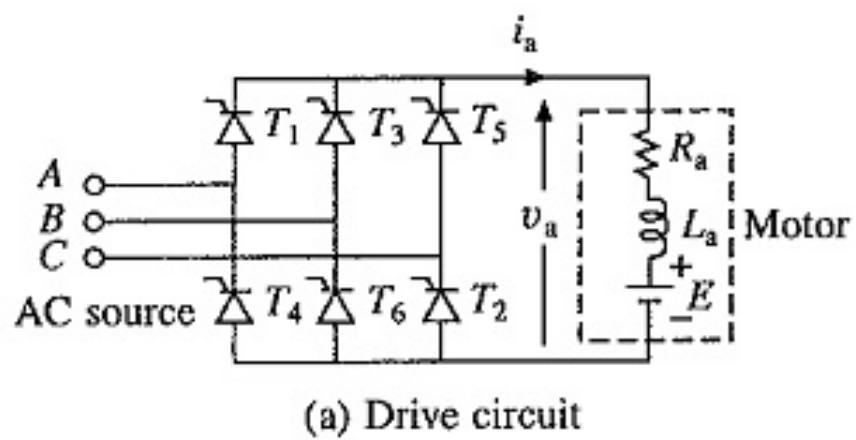
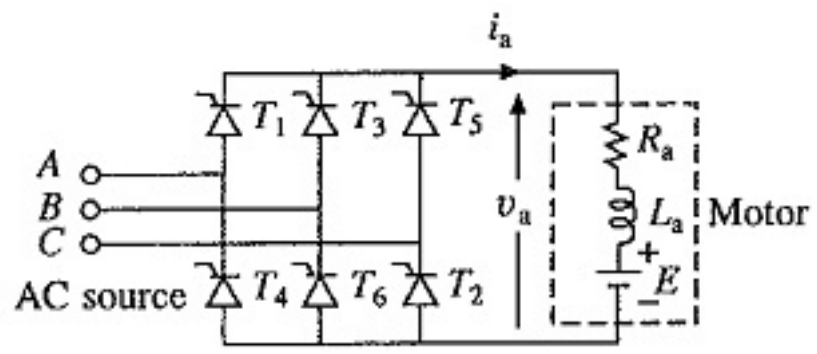
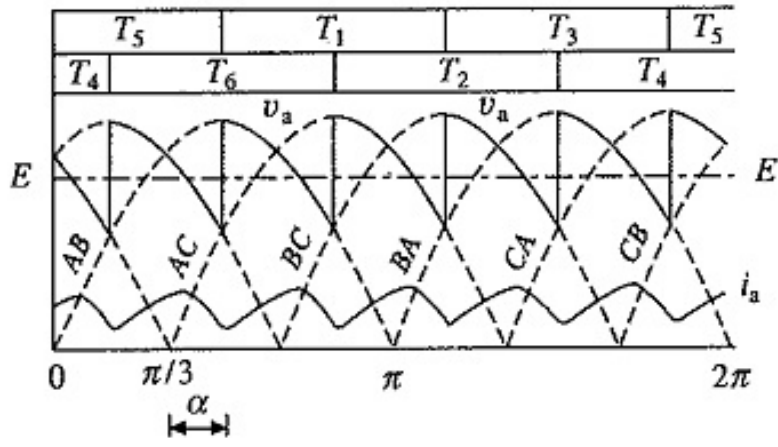


Fig. Three-phase fully-controlled converter control of separately excited motor

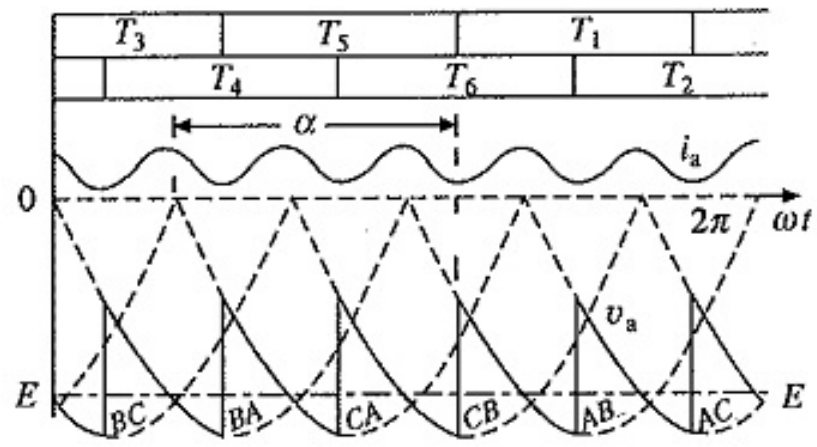
**Three phase Fully Controlled Rectifier Control of DC Separately Excited Motor:**



(a) Drive circuit



(b) Motoring operation,  $\alpha = 30^\circ$



(c) Braking operation  $\alpha \approx 140^\circ$

**Fig. Three-phase fully-controlled converter control of separately excited motor**

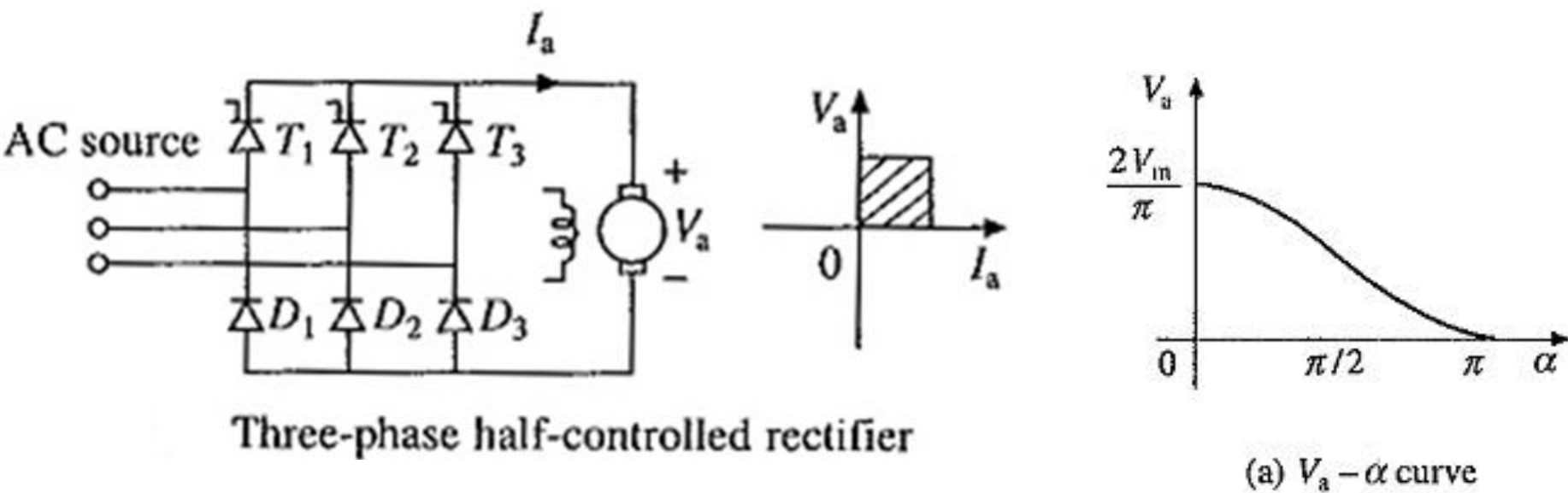
Three phase Fully Controlled Rectifier Control (6 pulse) fed separately excited dc motor drive is shown in Fig.(a). Thyristors are fired in the sequence of their numbers with a phase difference of  $60^\circ$  by gate pulses of  $120^\circ$  duration. Each thyristor conducts for  $120^\circ$ , and two thyristors conduct at a time—one from upper group (odd numbered thyristors) and the other from lower group (even numbered thyristors) applying respective line voltage to the motor



$$\begin{aligned}
 V_a &= \frac{3}{\pi} \int_{\alpha+\pi/3}^{\alpha+2\pi/3} V_m \sin \omega t d(\omega t) \\
 &= \frac{3}{\pi} V_m \cos \alpha
 \end{aligned}$$

Transfer of current from an outgoing to incoming thyristor can take place when the respective line voltage is of such a polarity that not only it forward biases the incoming thyristor, but also leads to the reverse biasing of the outgoing when incoming turns-on. Thus, firing angle for a thyristor is measured from the instant when the respective line voltage is zero and increasing. For example, the transfer of current from thyristor  $T_5$  to thyristor  $T_1$  can occur as long as the line voltage  $v_{AC}$  is positive. Hence, for thyristor  $T_1$ , firing angle  $\alpha$  is measured from the instant  $v_{AC} = 0$  and increases as shown in Figs. (b) and (c).

**Three Phase Half Controlled Rectifier Control of DC Separately Excited Motor:**



under continuous conduction  $V_a = \frac{3V_m}{2\pi} (1 + \cos \alpha)$

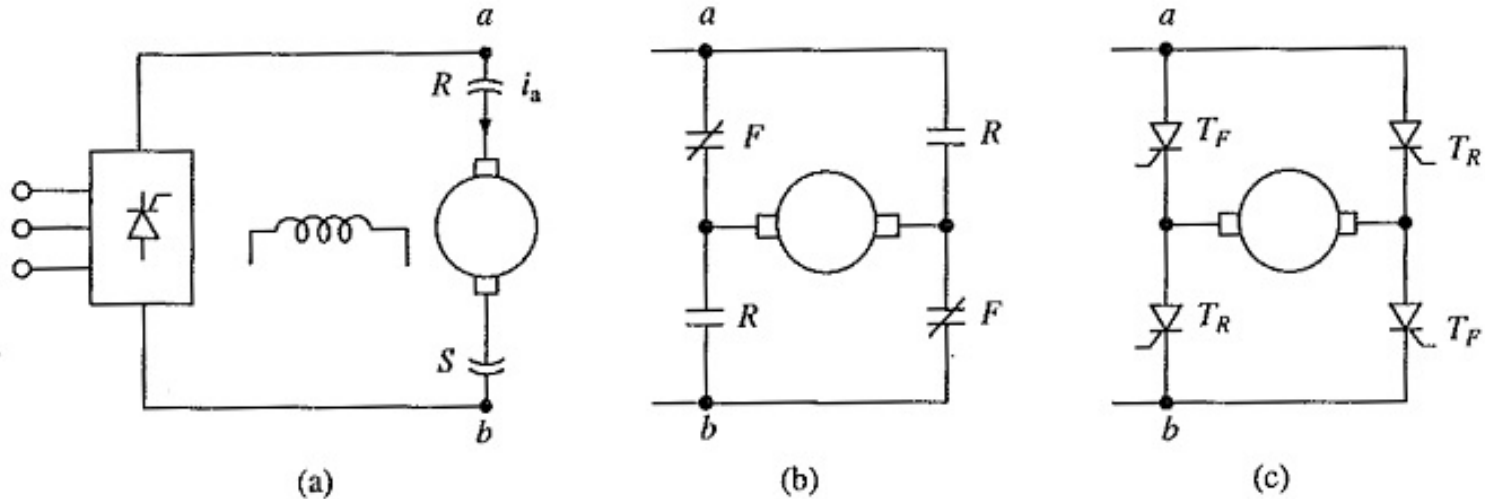
$$\omega_m = \frac{3V_m}{2\pi K} (1 + \cos \alpha) - \frac{R_a}{K^2} T$$

$V_a$  vs  $\alpha$  curve has same nature as shown in above Fig. (a). Consequently, drive operates only in quadrant I.

# Multiquadrant Operation of dc Separately Excited Motor Fed From Fully Controlled Rectifier

1. Single Phase Fully Controlled Rectifier with a reversing Switch
2. A Dual Converter Control of DC Separately Excited Motor
3. Four Quadrant Drive With Field Current Reversal

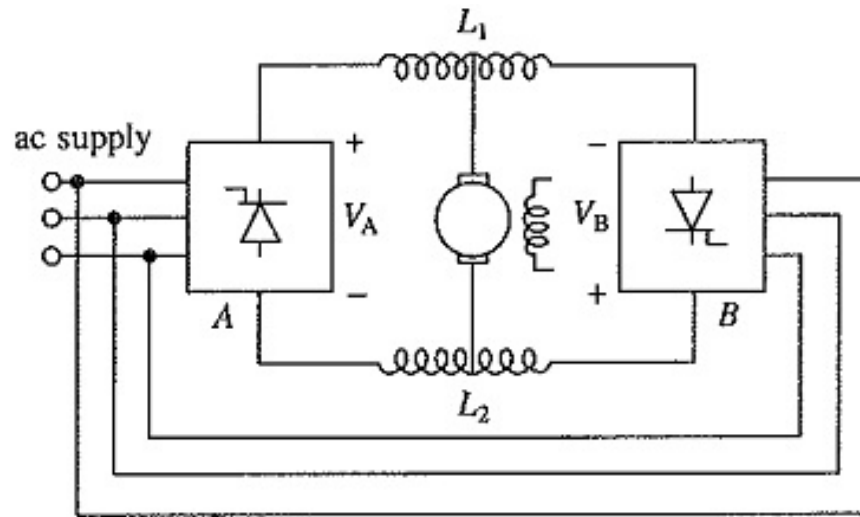
## Single Phase Fully Controlled Rectifier with a reversing Switch



**Fig. 1** Four quadrant drive employing single converter and a reversing switch

## A Dual Converter Control of DC Separately Excited Motor

A Dual Converter Control of DC Separately Excited Motor consists of two fully-controlled rectifiers connected in anti-parallel across the armature. For power ratings upto around 10 kW, single-phase fully-controlled rectifiers can be used. For higher ratings, three-phase fully controlled rectifiers are employed. Rectifier A, which provides positive motor current and voltage in either direction, allows motor control in quadrants I and IV, Rectifier B provides motor control in quadrants III and II, because it gives negative motor current and voltage in either direction.



**Fig.2** Dual converter control of dc separately excited motor. A and B are fully controlled rectifiers. Inductors  $L_1$  and  $L_2$  are used only with simultaneous control

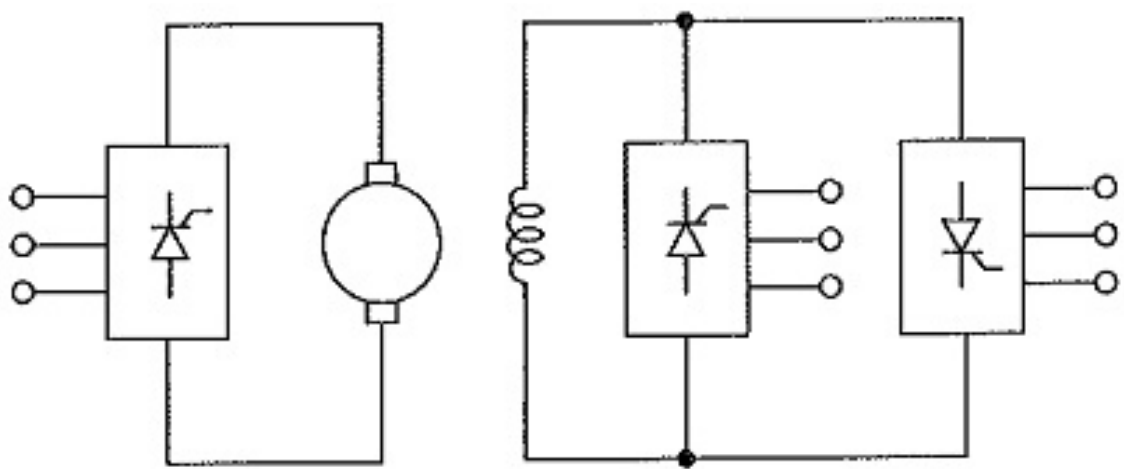
There are two methods of control for the Dual Converter Control of DC Separately Excited Motor:

(a) In simultaneous control both the rectifiers are controlled together. In order to avoid dc circulating current between rectifiers, they are operated to produce same de voltage across the motor terminals

(b) In non-simultaneous or non-circulating current control method, one rectifier is controlled at a time. Consequently, no circulating current flows and inductors  $L_1$  and  $L_2$  are not required. This eliminates losses associated with circulating current and weight and volume associated with inductors. But then discontinuous conduction occurs at light loads and control is rather complex.

# Four Quadrant Drive With Field Current Reversal

Four Quadrant Drive With Field Reversal as shown in below Fig. Armature is fed from a fully-controlled rectifier and the field from a dual converter so that field current can be reversed. With field current in one direction, the motor operates in quadrants I and IV. When field current is reverted, it operates in quadrants III and II. The dual converter operates with non-simultaneous control.



**Fig.3** Four quadrant drive with field reversal

The speed reversal is done as follows.

The armature rectifier firing angle is set at the highest value to force the armature current to zero and then firing pulses are withdrawn. The firing angle of the rectifier supplying the field is now set at the highest value. It operates as an inverter and the field current is forced to zero. After a suitable dead time, the second rectifier is activated at the lowest firing\_angle.

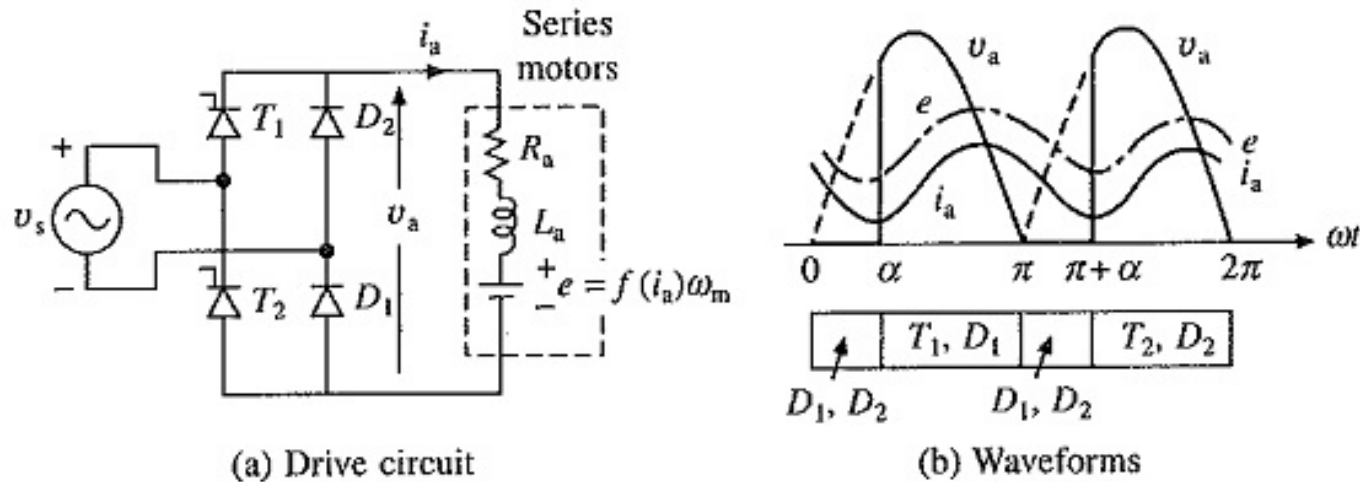
When the field current has nearly settled and the motor back emf has reversed, the firing pulses of the armature rectifier are released so as to set the firing angle at the highest value. Now onwards the current control loop adjust the firing angle continuously to brake and then accelerate the motor at a constant current to the desired speed in the reverse direction.



When speed control in wide range is required, field current is also controlled. In armature voltage control schemes of Figs. 1 and 2 , the field is then supplied by either a fully-controlled or a half-controlled rectifier. In the scheme of Fig. 3, dual converter is utilized for the control of field current

# Rectifier Control of DC Series Motor:

Single-phase controlled Rectifier Control of DC Series Motor are employed in traction. A single-phase half-controlled Rectifier Control of DC Series Motor is shown in Fig. (a)



**Fig. a** Single-phase half-controlled rectifier fed series motor

Motor operation is described by following equations for duty and freewheeling intervals respectively,

$$V_m \sin \omega t = R_a i_a + L_a \frac{di_a}{dt} + f(i_a) \omega_m, \quad \text{for } \alpha \leq \omega t \leq \pi \quad \dots(1)$$

$$0 = R_a i_a + L_a \frac{di_a}{dt} + f(i_a) \omega_m, \quad \text{for } \pi \leq \omega t \leq (\pi + \alpha) \quad \dots(2)$$

$$E_a = K_a \omega_m \quad \dots(3)$$

$$K_a = f(I_a) \quad \dots(4)$$

Since the drop across the inductance  $L_a$  due to dc component of armature current  $I_a$  is zero

$$V_a = E_a + I_a R_a$$

$$\omega_m = \frac{V_a - I_a R_a}{K_a} \quad \dots(5)$$

$$T = K_a I_a \quad \dots(6)$$

For continuous conduction,  $V_a$  for half-controlled rectifiers is given by Eqs

$$V_a = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t d(\omega t) = \frac{2V_m}{\pi} \cos \alpha \quad \dots(7)$$

For continuous conduction,  $V_a$  for fully-controlled single-phase rectifiers is given by Eqs.

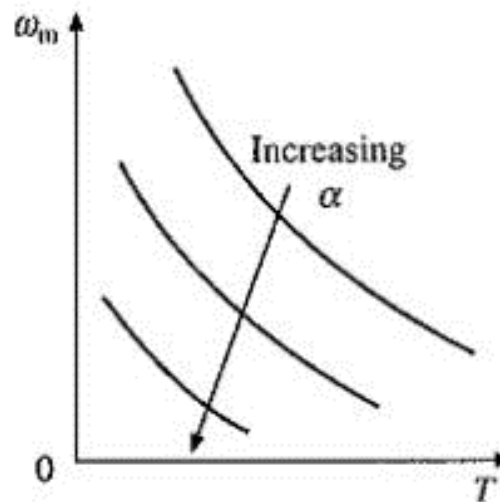
$$V_a = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t) = \frac{V_m}{\pi} (1 + \cos \alpha) \quad \dots(8)$$

Following sequence of steps are used to calculate speed-torque characteristic for a given  $\alpha$  taking into account non-linearity of the magnetic circuit:

A value is chosen for  $I_a$ . Corresponding value of  $K_a$  is obtained from the magnetization characteristic of the motor.

For the known value of  $\alpha$ , calculate  $V_a$  from Eq. (7) or (8), depending on the rectifier circuit used.  $\omega_m$  and  $T$  are obtained from Eqs. (5) and (6), respectively.

Nature of speed-torque characteristics for the drive of Fig. (a) is shown in Fig. 5.38.



**Fig. b** Speed torque curves of series motor fed from a controlled rectifier

# Control of Fractional hp Motors:

Low cost single-phase half-wave controlled rectifier of Fig.(a), employing a single thyristor, is commonly used for the control of fractional hp universal Motor, dc series and permanent-magnet dc motors. Such drives are employed in hand tools and small domestic appliances

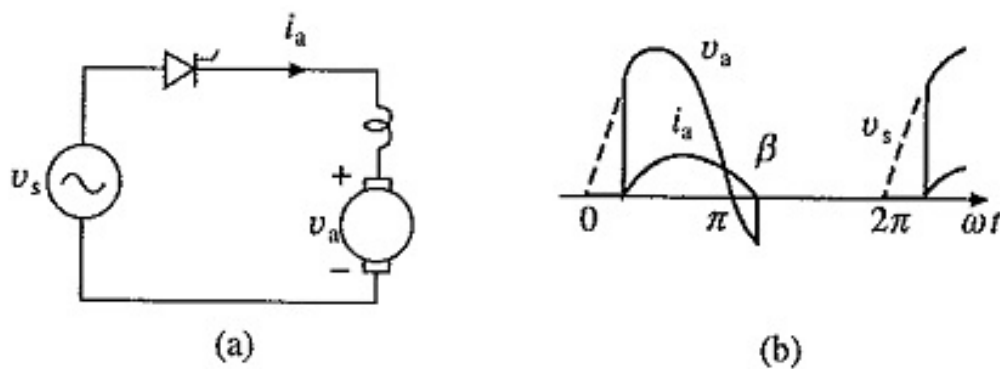


Fig. 5.39 Control of universal motor by a single thyristor

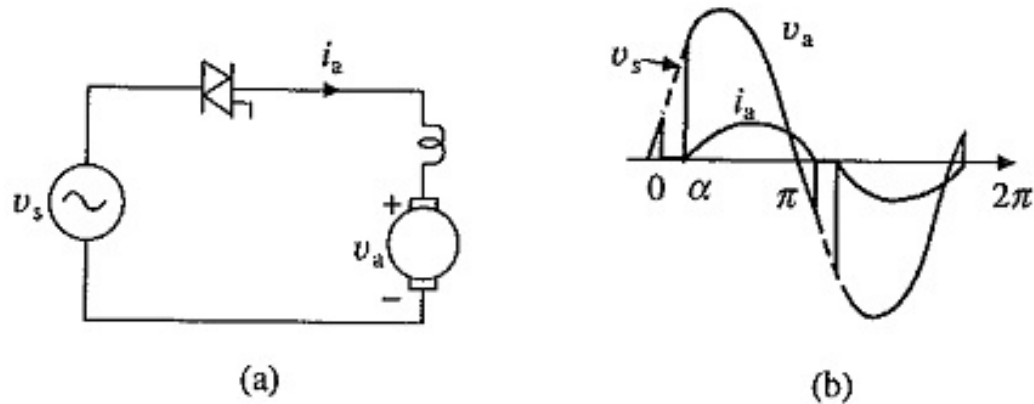


Fig. a Control of universal motor by an ac voltage controller

## **Drawbacks of Rectifier Fed DC Drives:**

Supply Harmonics, Power Factor and Ripple in Motor Current

Drawbacks of Rectifier Fed DC Drives are as follows:

### **1. Distortion of Supply:**

Source voltage and current distortions have several undesirable effects including interference with other loads connected to the source and radio frequency interference in communication equipment.

## 2. Low power factor:

Assuming sinusoidal supply voltage, power factor (PF) of a rectifier can be defined as

$$PF = \frac{\text{Real Power}}{\text{Apparent Power}} = \frac{V I_1 \cos \phi_1}{V I_{\text{rms}}}$$

$$PF = \frac{I_1}{I_{\text{rms}}} \cos \phi_1 = \mu \cos \phi_1$$

$\mu$  is called the distortion factor and  $\cos \Phi_1$  is the displacement factor

supply power factor is low when the drive operates at low speeds.

Pulse width modulated rectifiers are being built using insulated gate bipolar transistors (IGBT) and gate turn-off thyristors (GTO) as they have high power factor and low harmonic content in source current but then their efficiency is low because of high switching losses.

### **3. Ripple in Motor Current:**

The presence of harmonics increases both copper loss and core loss.

The motor output (power and torque) has to be restricted considerably below rated value in order to avoid thermal overloading and sparking at brushes.



## Chopper-Controlled DC Drives

The chopper converts the fixed DC voltage to variable DC voltage. Self-commutated devices (directly on or off devices via gate) like MOSFET, IGBT, power transistors, GTO and IGCT are used for making choppers because they can be commutated by low power control signal and do not need commutation circuit.

# Chopper Control of Separately Excited DC Motor:

**Motoring Control :** A transistor Chopper Control of Separately Excited DC Motor drive is shown in Fig.1

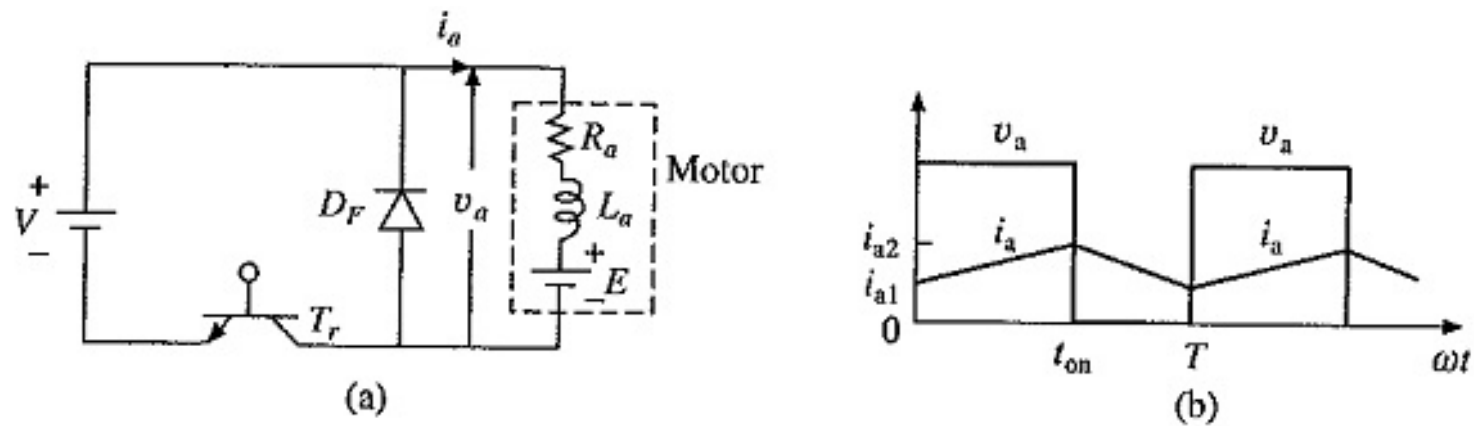


Fig. 1 Chopper control of separately excited motor

During on-period of the transistor,  $0 \leq t \leq t_{on}$ , the motor terminal voltage is  $V$ .

The operation is described by 
$$R_a i_a + L_a \frac{di_a}{dt} + E = V, \quad 0 \leq t \leq t_{on}$$

In this interval, armature current increases from  $i_{a1}$  to  $i_{a2}$ . Since motor is connected to the source during this interval, it is called **Duty Interval**.

## Chopper Control of Separately Excited DC Motor:

At  $t = t_{\text{on}}$ ,  $T_r$  is turned-off. Motor current freewheels through diode  $D_F$  and motor terminal voltage is zero during interval  $t_{\text{on}} \leq t \leq T$ . Motor operation during this interval, known as freewheeling interval, is described by

$$R_a i_a + L_a \frac{di_a}{dt} + E = 0, \quad t_{\text{on}} \leq t \leq T$$

Motor current decreases from  $i_{a2}$  to  $i_{a1}$  during this interval.

$$\delta = \frac{\text{Duty interval}}{T} = \frac{t_{\text{on}}}{T}$$

$$V_a = \frac{1}{T} \int_0^{t_{\text{on}}} V dt = \delta V$$

$$I_a = \frac{\delta V - E}{R_a}$$

$$\omega_m = \frac{\delta V}{K} - \frac{R_a}{K^2} T$$

**Regenerative Braking:**

Chopper Control of Separately Excited DC Motor for regenerative braking operation is shown in below Fig.

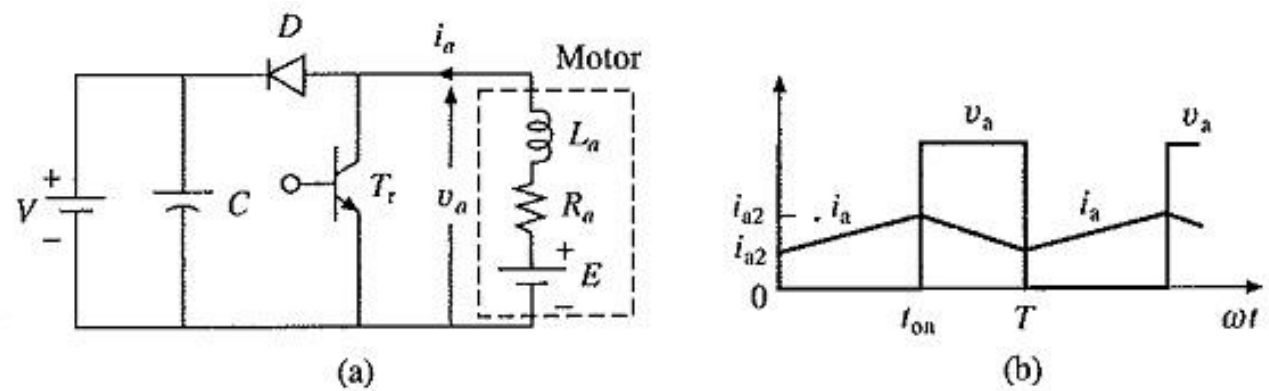


Fig. Regenerative braking of separately excited motor by chopper control

Transistor  $T_r$  is operated periodically with a period  $T$  and on-period of  $t_{on}$ . Waveforms of motor terminal voltage  $v_a$  and armature current  $i_a$  for continuous conduction are shown in Fig. (b).

Usually an external inductance is added to increase the value of  $L_a$ . When  $T_r$  is on,  $i_a$  increase from  $i_{a1}$  to  $i_{a2}$ .

$$\delta = \frac{\text{Duty interval}}{T} = \frac{T - t_{on}}{T}$$

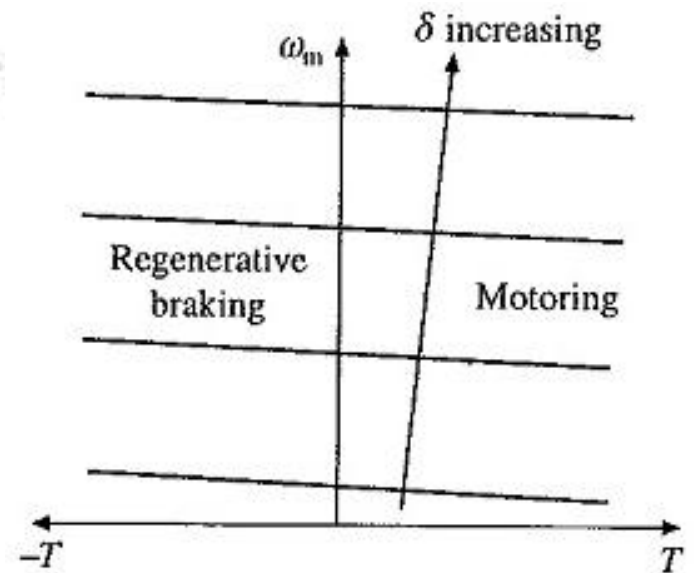
$$V_a = \frac{1}{T} \int_{t_{on}}^T V dt = \delta V$$

$$I_a = \frac{E - \delta V}{R_a}$$

Since  $I_a$  has reversed  $T = -KI_a$

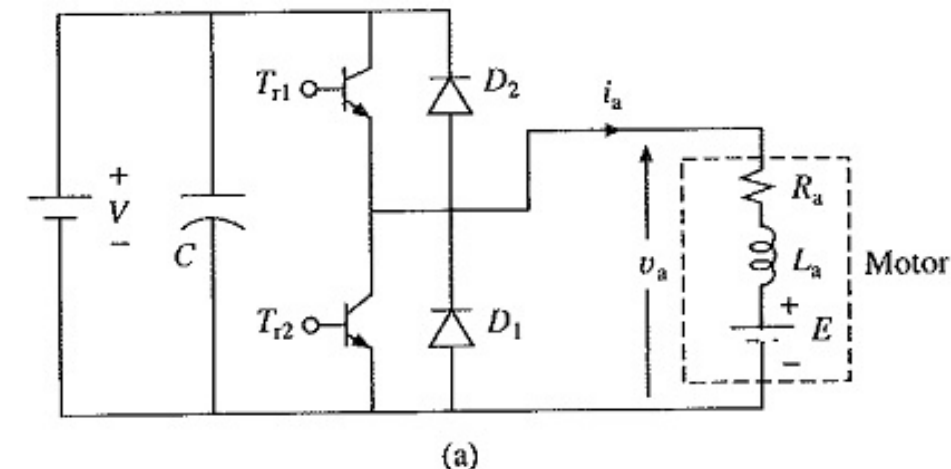
$$\omega_m = \frac{\delta V}{K} - \frac{R_a}{K^2} T$$

The nature of speed torque characteristic is shown in Fig



**Fig.** Speed torque curves of chopper controlled separately excited motor

# Motoring and Regenerative Braking:



$$V_a = \delta V$$

$$I_a = \frac{\delta V - E}{R_a}$$

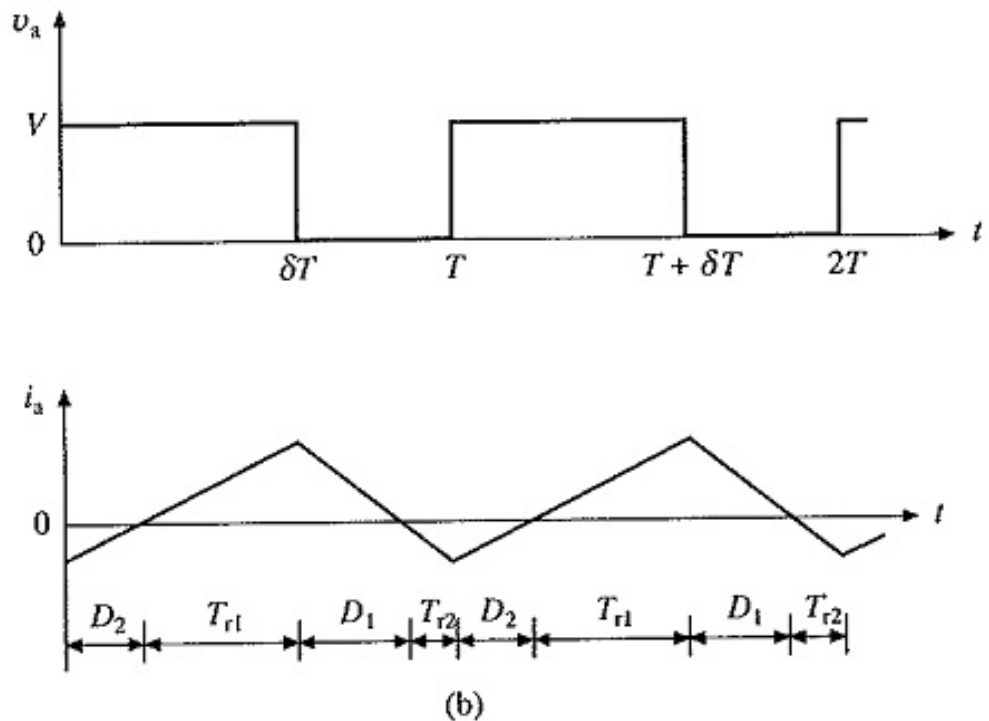


Fig. Chopper for forward motoring and braking control

Dynamic Braking:

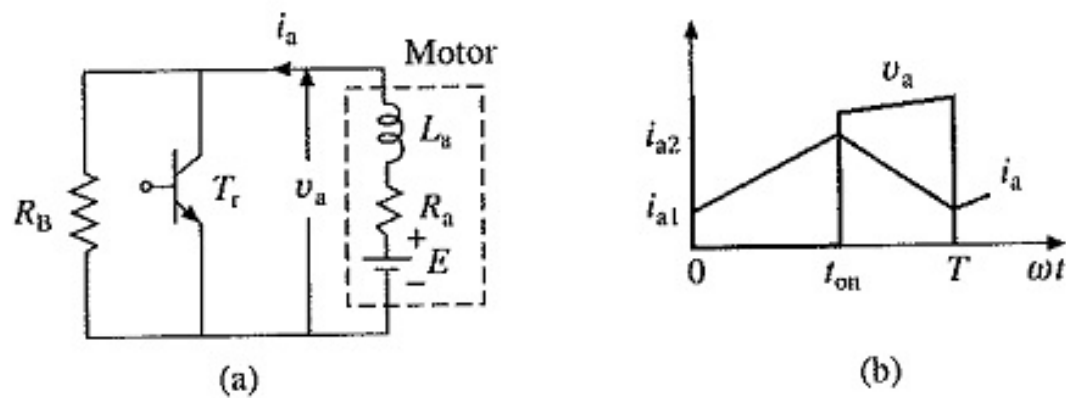


Fig. Dynamic braking of separately excited motor by chopper control

If  $i_a$  is assumed to be rippleless dc, then energy consumed  $E_N$  by  $R_B$  during a cycle of chopper operation is

$$E_N = I_a^2 R_B (T - t_{on})$$

Average power consumed by  $R_B$

$$P = \frac{E_N}{T} = I_a^2 R_B (1 - \delta)$$

Effective value of  $R_B$

$$R_{BE} = \frac{P}{I_a^2} = R_B (1 - \delta)$$

where

$$\delta = \frac{t_{on}}{T}$$

Chopper Control of Series Motor:

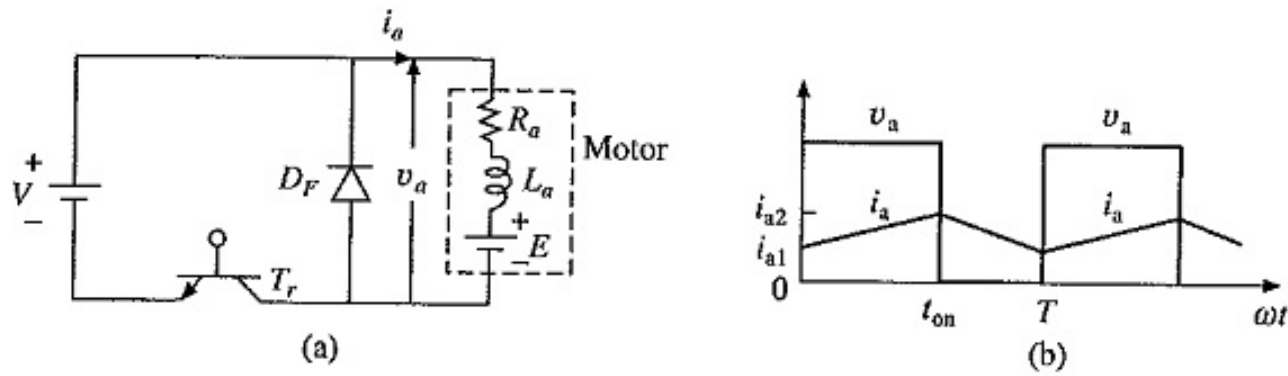


Fig. Chopper control of separately excited motor

$$V_a = \frac{1}{T} \int_0^{t_{on}} V dt = \delta V$$

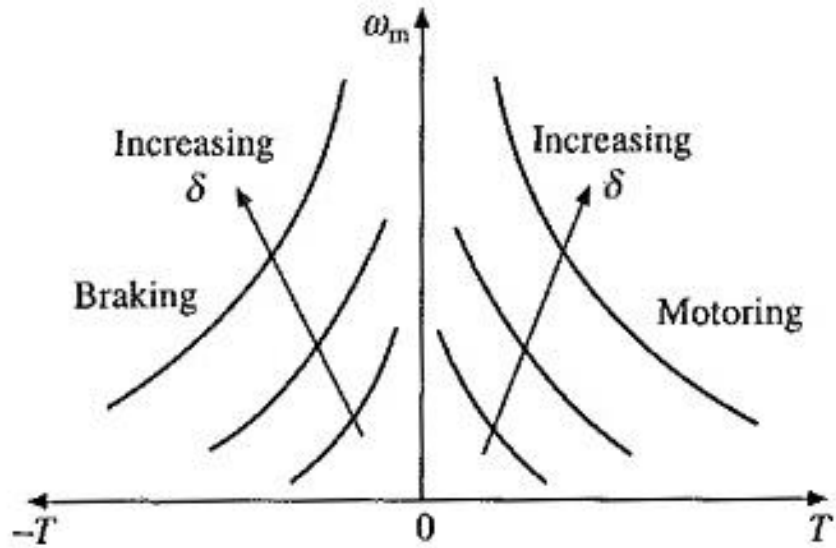


Fig. Motoring and regenerative braking characteristics of chopper controlled series motor



## Regenerative Braking:

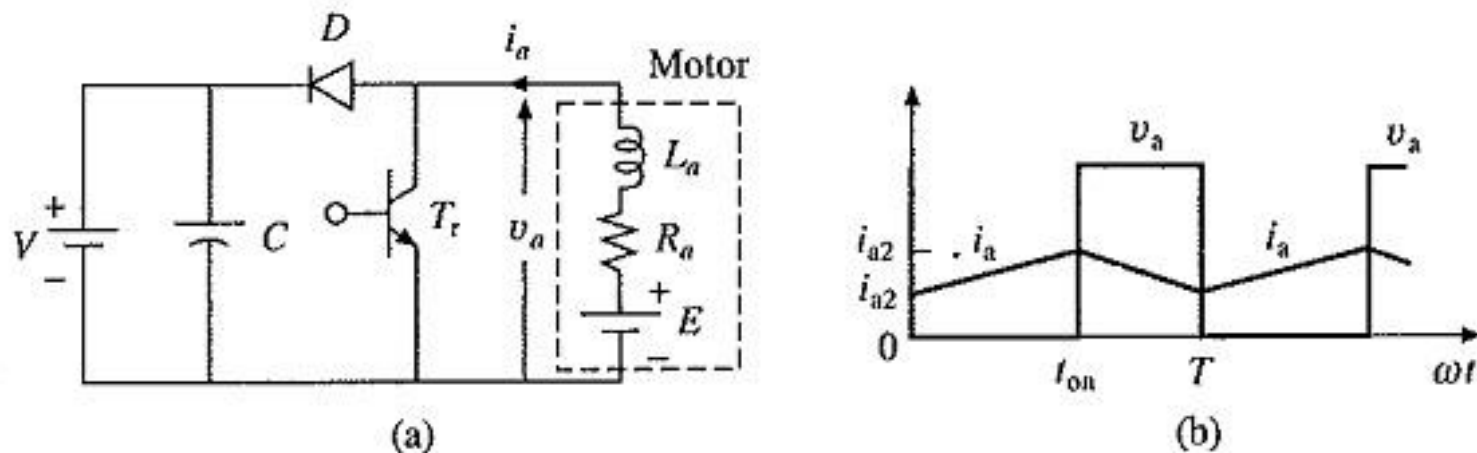


Fig. Regenerative braking of separately excited motor by chopper control

$$\omega_m = \frac{\delta V + I_a R_a}{K_a}$$

$$T = -K_a I_a$$

## Dynamic Braking:

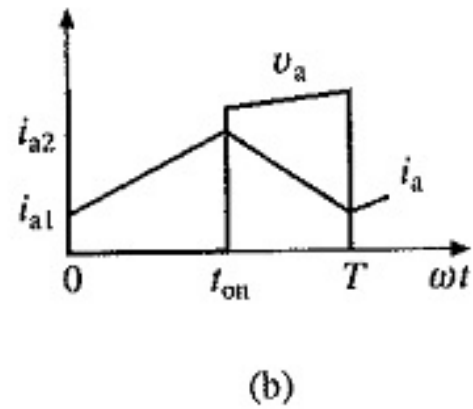
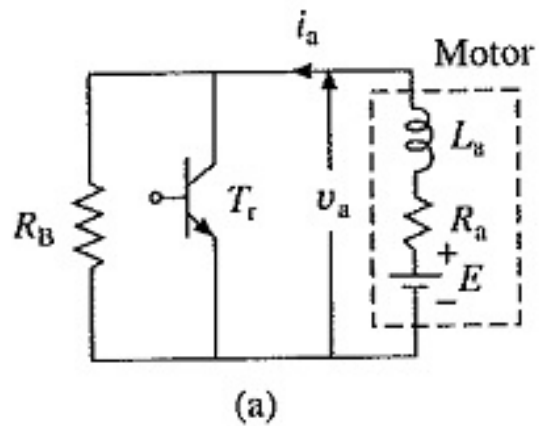


Fig. Dynamic braking of separately excited motor by chopper control

Converter rating and Closed Loop Speed Control of DC Motor:

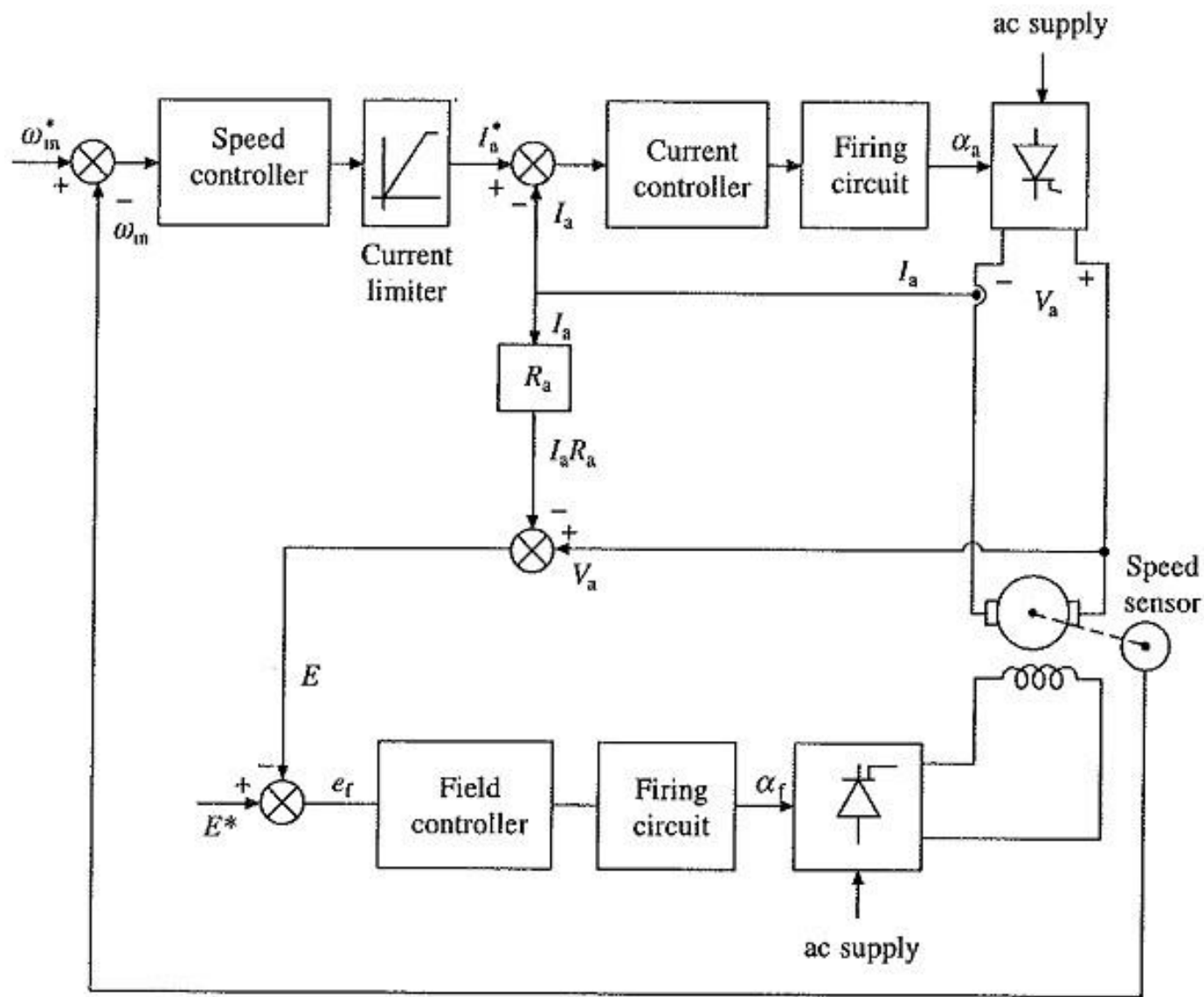


Fig. Closed-loop speed control scheme for control below and above base speed

## Assignment

A 220V, 1500 rpm, 50A, separately excited dc motor with armature resistance of  $0.5\Omega$  is fed from a 3 phase fully controlled rectifier. The available ac source is 440V, 50Hz. A star delta connected transformer is used to feed the armature so that the motor terminal voltage equals rated voltage when converter firing angle is zero.

- (i) Calculate the transformer turns ratio
- (ii) Firing angle when (a) motor is running at 1200 rpm and rated torque; (b) 800 rpm and twice the rated torque

Assume continuous conduction

For a 3 phase Fully controlled converter

$$\begin{aligned} V_a &= \frac{3}{\pi} \int_{\alpha+\pi/3}^{\alpha+2\pi/3} V_m \sin \omega t d(\omega t) \\ &= \frac{3}{\pi} V_m \cos \alpha \end{aligned}$$